WRITING TO LEARN STATISTICS IN AN ADVANCED PLACEMENT STATISTICS COURSE

by

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ABSTRACT

CHRISTIAN GLENN NORTHRUP. Writing to learn statistics in an Advanced Placement Statistics course. (Under the direction of DR. DAVID K. PUGALEE).

This study investigated the use of writing in a statistics classroom to learn if writing provided a rich description of problem-solving processes of students as they solved problems. Through analysis of 329 written samples provided by students, it was determined that writing provided a rich description of problem-solving processes and enabled teachers to find student mistakes easier. Requiring students to write in a statistics course provided a window into the problem-solving abilities of students. The researcher also concluded that he was better able to help students fix errors and misunderstandings since writing made them easier to find. This study also investigated if there any differences when analyzing written samples of students, using ratings from a rubric, for problem-solving processes as they solve problems. A Hierarchical Linear Modeling (HLM) procedure was used and found one statistically, significant difference between the problem-solving process of conceptual understanding and that of problem-solving ability, $t$ (1643) = -9.231, $p < .001$. This suggested that students received a significantly lower score for conceptual understanding compared to problem-solving ability when the researcher analyzed their work using a rubric designed by Pugalee (2005).

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| | |
|---|---|
| ACT | American College Test |
| AP | Advanced Placement |
| HLM | Hierarchical Linear Modeling |
| ICC | Intra-class Correlation |
| PSAT | Preliminary Scholastic Aptitude Test |
| SAT | Scholastic Aptitude Test |
| WTLM | Writing to Learn Mathematics |

# CHAPTER 1: INTRODUCTION

For the first time in history, with the advent of the Information Age, people across the world have easy access to statistics. Our society benefits greatly from the ability to appropriately interpret and understand statistics. Steffens and Fletcher (1999) state, "Deemed of more importance than the theory and mechanic of statistics is the analysis of the data, the interpretation of the data and communicating the findings" (p. 298). High school students who enroll in an Advanced Placement Statistics course gain the foundation necessary to help with interpreting and understanding statistics. In addition to understanding statistics, students are expected to learn to properly communicate their findings through a myriad of forms, including graphs, pie charts, histograms, and written paragraphs. Instruction including a focus on writing is critical to developing these skills.

While Friedman (2000) supports the use of writing by students in a mathematics classroom to demonstrate their knowledge of material being learned, there are dissenting opinions. In my discussions with colleagues, it became clear that many statistics teachers did not see the value of using writing to help deepen the understanding of statistical concepts, and writing was therefore not being utilized as a teaching tool. This stance is perplexing as statistics requires students to make inferences and write solutions after solving problems. To support the use of writing in a statistics course, this study examined a link between two important aspects of mathematics, writing (Pugalee, 2005) and problem solving (Polya, 1962).

Writing in a mathematics classroom can play a vital role in helping students gain a better understanding of topics taught in a statistics classroom. According to National Council of Teachers of Mathematics [NCTM] (2000), writing in mathematics requires students to clearly articulate their ideas and reflect on concepts being learned. Vygotsky (1987) believes that writing purposefully requires one to analyze thoughts and words to gain a complete understanding. Others assert that writing is an instrument that can assist students in learning new concepts (Pugalee, 2005; Pugalee, 2004; Pugalee, 2001; Rose, 1990; Shield & Galbraith, 1998). As such, writing often involves a reflective process (Baxter, Woodward, & Olson, 2005; Ediger, 2006) that allows students to think about their approach to solving a problem. Student writing also provides teachers with insight into student learning (Langer & Applebee, 1987; Waywood, 1994) and gives educators another tool to see connections between writing, problem solving, and metacognition as students solve mathematics problems (Pugalee, 2001).

The *Principles and Standards for School Mathematic*s set forth by the National Council of Teachers of Mathematics (2000) makes clear the importance of problem solving in mathematics classrooms, identifying it as one of five key process skills. Many researchers have studied problem solving, recognized its significance in a mathematics classroom, and recommended further research (Lester, 1994; McLeod, 1989; Polya, 1962; Schoenfeld, 1985; Silver, 1985). For the purpose of this study, the characteristics and processes of problem solving that will be analyzed and discussed have been described by Pugalee (2005) and include conceptual understanding, procedural understanding, mathematical reasoning, mathematical content, and problem-solving ability.

The processes of writing and problem solving in a mathematics classroom have been linked (Flower & Hayes, 1983; Pugalee, 2005; Pugalee, 2001) though there is little work investigating this relationship in a statistics classroom. This study examined the link between writing and problem solving in a statistics classroom. The participants were high school students currently enrolled in an Advanced Placement Statistics course. This study seeks to contribute to understanding how writing and problem solving play a key role in developing students' statistical understanding.

## Research Questions

Qualitative Question:

How do students' problem-solving processes through writing provide a rich description as they solve problems in a high school Advanced Placement Statistics course?

Sub-questions:

- How can one describe the problem-solving ability in students' writings as they solve problems in a statistics course?
- How can one describe the conceptual understanding in students' writings as they solve problems in a statistics course?
- How can one describe the procedural understanding in students' writings as they solve problems in a statistics course?
- How can one describe the mathematical content in students' writings as they solve problems in a statistics course?
- How can one describe the mathematical reasoning in students' writings as they solve problems in a statistics course?
- How do the students' writings improve over time?

- How can one describe problem posing by students as they solve problems in a statistics course?

Quantitative Question:

Are there any differences between the problem-solving processes of problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning when analyzing written samples of students using ratings from a rubric?

# CHAPTER 2: REVIEW OF LITERATURE

"Mathematical literacy is best viewed as a set of thinking processes that has certain characteristics" (Pugalee, 2005, p.xvi). These thinking processes include representing, manipulating, reasoning, and problem solving (Pugalee, 1999). Writing can help assist students with these processes while enabling educators to better understand students' problem-solving processes.

The literature review will be divided into four sections: 1) Writing and Learning, 2) Writing and Statistics, 3) Problem Solving, and 4) Writing and Problem Solving. The literature review is divided into these four sections to show links of writing with learning, statistics, and problem solving.

## Writing and Learning

### Writing

Vygotsky (1987) believed that writing purposely requires one to process concepts analytically to gain understanding. He also believed that writing compels an individual to organize thoughts in a coherent and rational way, forming connections between new information with previously learned information. By making these connections through writing, it allows a student to gain a better understanding of a topic (Pugalee, 2001). Langer and Applebee (1987) also believed that the use of writing allowed students to process and reflect on a particular concept. "Thinking skills are taught best when related to some content, the argument goes, and writing provides a particularly welcome context for thinking deeply about such content" (Langer and Applebee, p. 1). Vygotsky (1962) viewed both thought and language as being linked through conceptual understanding. The link can be seen through verbal discourse. Pugalee (1999) argued that "not only is verbal discourse important in the development of mathematical literacy: writing has also been shown to create an environment that supports the type of metacognitive thinking that, in turn, supports mathematical reasoning" (p. 21).

Writing is a learning tool that allows a student to transform concepts rather than reproducing concepts (Boscolo & Mason, 2001; Jurdak & Abu Zein, 1998). Mere reproduction of concepts does not ensure students actually understand the subject matter. The process of transforming concepts involves a reflective process that enables students to better understand the concepts. Writing often involves a reflective process (Baxter, Woodward, & Olson, 2005; Ediger, 2006) that drives students to think about their approach to solving a problem, an important skill in mathematics.

Others in the field of mathematics education believe that writing can be useful in the mathematics classroom (Pugalee, 2004; Pugalee, 2001; Rose, 1990; Shield & Galbraith, 1998). However, it is also a useful tool for teachers to use in assessing students. For example, writing has been shown to enable teachers to gain insight to mathematical learning over a period of time and better discuss with parents a facet of their child's learning not otherwise known (Fortescue, 1994; Gordon & MacInnis, 1993; Langer & Applebee, 1987; Waywood, 1994).

## Writing in Mathematics

Writing in mathematics helps students realize mathematics is significant (Pugalee, 1997; Rose, 1990). Porter and Masingila (2000) investigated the effects of writing to learn in mathematics (WTLM), which they defined as "writing that involves articulating and explaining mathematical ideas for the purpose of deepening one's understandings" (p. 166). They specifically probed the role that writing plays as students develop knowledge in calculus. The authors conducted the research to find if WTLM improves conceptual understanding and affect students' abilities to perform routine skills and procedures. They concluded that the results supported the premise that writing helps students better understand calculus concepts.

Abel and Abel (1998) stated, "Since the writing process can be a medium for learning mathematics, writing should be an integral part of every mathematics class" (p. 155). However, this statement seems to be a bit of a conclusory overreach the way in which it is written. For instance, an abacus is a medium in which to learn mathematics. Should it be an integral part of every mathematics class? Still, the authors concluded that students performed better in mathematics classrooms when using writing as a tool and also stated that teachers of all subjects need to be aware of the benefits that writing can have in their classrooms, including a mathematics classroom. As students write more, they will become better writers (Miller & England, 1989; Connolly & Vilardi, 1989; Kenney, 1992).

## Challenges of Writing in Mathematics

Finding appropriate writing activities for mathematics can be challenging. Shield and Galbraith (1998) conducted a study involving students in the eighth grade, in which they analyzed expository writing samples of students, compared the writing style to the style used by textbooks, and investigated the effects of writing on comprehension of mathematical concepts. The authors discovered that the majority of written samples produced by the students matched the writing style of textbooks. Students imitated the style of writing in textbooks because that had been their only exposure to writing in mathematics. Shield and Galbraith concluded that writing in mathematics will not lead to a greater understanding of concepts unless students are able to write in a manner that "promotes a higher level of thinking" (p. 45). Students were constrained if they used a textbook style of writing as a guide to clarify their thoughts regarding mathematical concepts or processes. Shield and Galbraith did not discuss ways that enabled students to write in a manner that deepen understanding of concepts, but they did suggest that "major shifts in teaching practices and textbooks to which students are exposed throughout their school lives" (p. 45) would need to occur before any meaningful writing can take place in mathematics classrooms.

An implication of their study relates to the design of the writing tasks that students are expected to complete. Students are likely to revert to the only way that they know how to write mathematics, the way that they have seen it in textbooks since their first day in a mathematics class. In order to help educators remove these constraints for students, appropriate writing activities need to be assigned to promote the higher levels of thinking among students (Shield & Galbraith, 1998).

Knowing the importance of the design of writing prompts for students in mathematics led me to search for literature involving good designs. It was critical to find writing prompts that were conducive to deepen the learning of concepts for my study. Pugalee (2005) supplied readers with an extensive list of writing prompts to use in a mathematical setting. My study will use writing prompts from Pugalee's list to prevent students from imitating writing seen in textbooks as in the study by Shield and Galbraith.

Understanding Mathematical Concepts

Writing in a mathematics classroom can help students convey their understanding of newly-learned mathematical concepts when they are not comfortable doing so verbally. Clarke, Waywood, and Stephens (1993) believed that students need to share their understanding of mathematical concepts with others to truly understand it. In a typical mathematics classroom, one form of communication occurs through oral discourse. Not all students are comfortable sharing their thoughts and ideas verbally. Baxter, Woodward, and Olson (2005) suggest that not all students benefit the greatest from verbal dialogue in a mathematics classroom for various reasons, including anxiety. Other forms of communication, like writing, benefits students. Pugalee (1995, 1997) has shown that writing supports mathematical reasoning. He stated that writing can be linked to increased metacognition for students who write as they solve problems in mathematics. Writing allows students to demonstrate their understanding of concepts when they struggle to do so through oral discussion.

Baxter, Woodward, and Olson (2005) conducted a study involving twenty-eight seventh-grade students of lower-achieving ability. Four students in the study qualified for special education assistance. The students were expected to write in a journal at least once a week for the entire school year. The authors looked at journal writing (about feelings and opinions) and expository writing (math journals intended to explain). The authors viewed expository writing as a way to permit students "to write about their mathematical ideas and reasoning" (p. 121). Baxter, Woodward, and Olson acknowledge that "students write in most mathematics classes, but they typically write numerical answers to problems or symbols to show the computational steps they used to arrive at a particular answer" (p. 120). The study found that the students were unable to communicate their understanding of concepts verbally to their peers and teachers. The results also showed that some students (particularly females) believed that writing helped make more connections with the concepts. Baxter, Woodward, and Olson concluded the "writing provided an alternative strategy for three of the four target students to communicate their mathematical thinking" (p. 130). The authors believed that students gained a deeper understanding of concepts of mathematical ideas when they communicated their thinking through writing.

Miller (1982) asserted that writing improves a student's understanding in any subject that they are learning. King (1982) also concluded that students are likely to understand a mathematics topic better when having to write about it because it forces them to individually describe it without the help of others. Students who write in mathematics classrooms must organize their thoughts about the concept before actually writing about it, which in turns, allows for a deeper understanding (Elliott, 1996; Johnson, 1983).

These studies demonstrated that writing can help deepen the understanding of mathematical concepts for students. This supports a premise of my study that writing can be used as a tool to deepen the understanding of statistical concepts as they solve problems. My study built on this research and involved higher-achieving students in high school, whereas Baxter, Woodward, and Olson (2005) studied lower-achieving students in middle school. In addition, it examined in a statistics classroom the assertion by Miller (1982) that writing improves a student's understanding in any subject.

Possible Limitations of Writing in Mathematics

Baxter, Woodward, and Olson (2005) warn readers that the use of writing in a mathematical classroom has its limitations. While students are more likely to participate and express their feelings and mathematical thinking when writing compared to oral discussions, the primary limitation from a teacher's perspective is the amount of time it takes to read and respond to students concerning their writing activities. Perhaps the amount of time a teacher takes to read and respond can be decreased if students learn to write well, which will prevent teachers from having to search for pertinent information in student writing.

There are methods to combat the concerns raised by Baxter, Woodward, and Olson (2005) regarding disadvantages to the use of writing in a mathematics classroom. Pugalee (2005) listed five ways to manage feedback to students, which are:

1. Identify key strengths and weaknesses and address these with the entire class.
2. While monitoring students' work that involves writing, make comments and ask questions to guide writing as students are working on their tasks.
3. Use examples of good writing to show the entire class as a way of reinforcing and developing common performance expectations about written products.
4. Use peer and group assessment. Students can effectively identify and specify strengths and weaknesses in writing.
5. Limit written comments and focus questions or comments so they will guide writing (p. 21).

When discussing the use of writing as an aid to learning mathematics with fellow educators, I have been met with resistance. Managing feedback is a concern for teachers that are hesitant to employ writing in their classrooms. It was initially a fear of mine, as well. Using some of the ways to manage feedback discussed by Pugalee (2005), I was able to save a considerable amount of time scoring papers and focusing on comments that were beneficial for students. A couple of methods used for my study to save time were not looking at spelling and expecting only basic grammar to be used by students. For instance, if a student used a semicolon instead of a comma, it was overlooked. This helped keep the focus on the learning of statistics.

Journaling

Burns and Silbey (2000) discussed how writing in journals can improve learning in mathematics. The authors recommended four strategies that can motivate students to write in a mathematics classroom: 1) problem solving, 2) process prompts, 3) language experience, and 4) class discussion. Burns and Silbey also described each of the four strategies. They envision students writing in journals as they solve problems. They gave an example of students being asked “to write why a square is a special kind of rectangle” (p. 19). Process prompts are to help students initiate the task of writing, possibly having them reflect on their processes. When students are struggling with a problem, teachers should “encourage them to explain their thinking to you” (p. 19). After they speak two sentences, have them write those sentences and read them aloud. Once this is completed, the teacher may want to repeat this process. The final strategy, class discussion, involves students sharing how they completed a problem and making revisions to their writings afterwards. When students write in a mathematics classroom, it not only benefits students but teachers also. The process of writing in mathematics allows teachers to better understand the concepts that students truly understand and to address concepts that students find difficult.

## Conclusion

There are three points that are vital to my study. First, writing requires students to process concepts in a way that helps them understand those concepts (Vygotsky, 1987). Second, writing plays a major role in a mathematics classroom (Abel and Abel, 1998). Finally, writing involves a reflective process (Baxter, Woodward, & Olson, 2005) and can promote a higher level of thinking (Shield & Galbraith, 1998).

Research has linked writing with learning (Baxter, Woodward, & Olson, 2005; Ediger, 2006; Pugalee, 2005; Pugalee, 2004; Pugalee, 2001; Rose, 1990; Shield & Galbraith, 1998; Vygotsky, 1987; Waywood, 1994). In addition, writing to learn mathematics has been shown to be significant for students' learning (Gibson & Thomas, 2005; O'Connell, Beamon, Beyea, Denvir, Dowdall, Friedland, & Ward, 2005; Porter and Masingila, 2000) because it enables students to construct mathematics through representations, discussions, and investigations (Countryman, 1993). The use of writing as a learning tool permits students the chance to transform ideas rather than reproduce ideas (Boscolo & Mason, 2001; Jurdak & Abu Zein, 1998; Pugalee, 2005), which can lead to a deeper understanding of learned concepts.

## Writing and Statistics

The need to study practices that will help students better understand concepts is evidenced by the way that statistics assessment has evolved. Jolliffe (2007) discussed the numerous "changes over the last thirty to forty years in the way that statistics is taught and assessed" (p. 1). Many years ago on tests, students were expected to compute concepts "by hand" (p. 1) and solve problems they had already seen. With the use of technology, such as statistical packages on graphing calculators, becoming more prevalent in statistics education, this has allowed teachers to spend more time on analysis of data and interpretation in the context of the problem. While such emphasis may lend itself to writing, Jolliffe argued that writing assessments will generally appeal to students whose strengths lie in other subjects that are not directly related to statistics or mathematics. Jolliffe also argued that there are other reasons to compel students to write in statistics, believing that it will help them communicate with non-statisticians.

Other studies argue that writing helps students communicate and understand statistics (Jolliffe, 2007; Smith, 1998). It is also easier to identify common misconceptions of statistics when reading students' writing as they solve problems in statistics. In this manner, writing benefits teachers and students. It allows teachers the opportunity to find student mistakes and help the students correct those mistakes. Garfield, Hogg, Schau, and Whittinghill (2002) conducted a case study of statistics instructors. She asked the instructors how their course differed from a traditional course of statistics. One response that she received, "I teach statistics as a language course, and try to help the students develop literacy about statistics" (p. 5). This supports the fact that there are some statistics teachers who already value writing as a tool in their classroom.

While Pugalee (2005) argued that developing a mathematical literacy for classrooms should be a goal of all mathematics educators, Parke (2008) conducted research that supports how written and verbal communication in a statistics classroom can improve reasoning and understanding of concepts. Her research was completed using college students as participants and it spanned the whole semester. The students were expected to write about their results when completing hypothesis tests, sometimes trying to explain their results to people with a limited background in statistics. Smith

(1998) also argued that writing has its place in a statistics classroom. He conducted a semester-long study that had students complete projects requiring either oral or written reports. Student tests scores increased significantly, lending credence that statistics teachers should require students to write. An issue with Smith's work centers on the lack of frequency of writing taking place. He required students to complete only two projects where writing reports were involved. My study was writing intensive, requiring students to write three times a week when solving problems.

Garfield, Hogg, Schau, & Whittinghill (2002) wrote about results of a survey of teachers of a first statistics course. Some teachers suggested that statistics should be taught as a language course and that students should write in journals about coursework and reactions to problems they solved. Garfield (1994) also wrote about how assessment is evolving and tests are requiring students to apply knowledge to real-world problems. For students to apply learned concepts to real-world problems, they need to have a deeper understanding of the concepts. My study investigated if writing could aid and deepen the understanding of statistics. While the process of writing and its importance in a statistics classroom has also been connected through research (Garfield, Hogg, Schau, & Whittinghill, 2002; Jolliffe, 2007; Smith, 1998), more research is needed to develop a better understanding of writing and learning in statistics.

## Problem Solving

Problem solving is an important component when learning mathematics and statistics. The manner in which students solve problems can help students gain a better understanding of concepts that are being utilized in the problem. The *Principles and Standards for School Mathematics* set forth by the National Council of Teachers of

Mathematics (2000) list problem solving as one of five process skills expected of students. Problem solving is listed as one of the important skills for students as it will help students develop strategies that will help them deal with and attempt real-world problems. Problem solving has drawn considerable attention from researchers over the years and is a field of high interest among educators today.

The field of problem solving is more than just investigating different methods that students use when solving problems. Lester (1994) stated the four main areas in problem-solving where significant gains in research have been made are 1) determinants of problem difficulty, 2) distinctions between good and poor problem solvers, 3) attention to problem-solving instruction, and 4) the study of metacognition in problem-solving (p. 663). He discussed shared beliefs that the determinants of problem difficulty involve the "traits, dispositions, and experiential background" of the problem solvers (p. 665). For instance, a student who does not watch or play sports may have a difficult time solving a baseball problem involving slugging percentage. My study used this belief when requiring students to write problems using statistical concepts. This enabled participants to relate it to something in which they are familiar.

What separates a good problem solver from a bad problem solver? Lester (1994) pointed to five characteristics that separate "good" problem solvers from "bad" problem solvers. One characteristic that was fascinating is that "good" problem solvers focus on structural features of a problem when "bad" problem solvers focus on surface features (p. 665). Another point of problem-solving that Lester discusses is how many problem-solving programs are designed not on research but on folklore (p. 665). He claimed that the mainstream practice of "teaching students about problem-solving strategies and heuristics does little to improve their abilities to solve problems in general" (p. 666). While my study investigated student problem-solving processes as students wrote their solutions and problem posing, it is important to know if problem solving should be modeled. For my study, writing was modeled for students but problem-solving strategies were not modeled. Through writing, students were able to provide a description of their process for the researcher.

An additional point that Lester (1994) discussed was the study of metacognition and problem-solving. He stated that "teaching students to be more aware of their cognitions should take the place in the learning of specific mathematical concepts" (p. 667). Metacognition and problem solving are connected (Cai, 1994; Lester, 1994; McLeod, 1989; Pugalee, 2001; Schoenfeld; 1985) and writing helps students become aware of this connection (Pugalee, 2001). Metacognition will be discussed more later in this chapter.

Problem solving often requires students to use multiple strategies, including problem posing (Silver, 1994), which have become of interests to researchers over the past 20 years. Problem posing is the main strategy that will be investigated when analyzing the data for this study. Students in my study were asked to pose problems in a similar manner in which English (1997) had her participants pose problems using a given number or concept and Silver (1994) had participants doing post-solution problem posing. This will be discussed in the next section.

## Problem Posing

Many different areas of problem solving, such as heuristics (Polya, 1945, 1962) and problem posing (Silver, 1994), are being investigated as the research in mathematics education has grown (Schoenfeld, 1994). Specific forms of problem posing permit students to solve problems in a different manner (Silver, 1994). One form of problem posing requires students to reformulate the mathematics problem to make it easier to solve. Silver argues that students can be stimulated to solve a problem by answering one question, “How can I formulate this problem so that it can be solved?” (p. 20). If a teacher requires a student to answer this question in written form in a statistics course, then it will provide insight as to how students solve problems and what they focus on when reading that specific problem. Silver (1994) also believed that students can benefit from post-solution problem posing but warned the reader that further research was warranted.

In another form of problem posing, students are given a number and asked to pose a problem. English (1997) conducted a study of fifth-grade students and their problem-posing abilities. In her study, there were five themes or areas of problem solving. One of the themes, creating new problems from given problem components, was a process that contained problems that enabled students to choose a problem component and create their own problem. She discussed how students were to pose a problem only given a numerical answer (p. 191) or a verbal statement (p. 192). English (1997) concluded that the students who took part in the 10-week program seemed “to show substantial development in each of the program components, in contrast to those who did not participate” (p. 183). While the participants showed an “increase in the range of problems they would like to solve” (p. 209), the students still showed a dislike for nonroutine (novel) problems.

This supports my study by recognizing that students have the capability to problem pose through the use of writing. My study investigated the use of problem posing in a statistics classroom by having students solve a problem involving a statistical concept, followed by having them write a problem using that same concept in a different context. It is a similar approach to that of English, with a couple of changes, primarily that students solved a problem before posing one.

Problem posing is regarded as an important method to solving problems. Silver (1994) discussed the research conducted by Connor and Hawkins that concluded "having students generate their own problems improved their ability to apply arithmetic concepts and skills in solving problems" (p. 23). Silver also discussed how Koenker's work in 1958 "included problem posing as one of 20 ways to help students improve their problem solving" (p. 23). The significance of problem posing is further evidenced when Cifarelli and Sheets (2009) discussed that "posing problems is viewed by many as a useful classroom activity that may help nurture the mathematical thinking, and particularly, the problem solving actions of students" (p. 245).

## Writing and Problem Solving

Some researchers have connected the processes of problem solving and writing (Liljedahl, 2006; Pugalee, 2005; Pugalee, 2004; Pugalee 2001). Writing improved the process of problem solving for students (Ford, 1990; Johnson, 1983). It enabled students the opportunity to articulate their thoughts, which can lead to a deeper understanding of concepts being learned. One method for promoting writing is journaling. Liljedahl (2006) believes that persona-based journaling will help create "more representative journaling" of students' problem-solving processes (p. 65). He states that "problem solving is a process in that incorporates not only the logical processes of inductive and deductive reasoning, but also the extra-logical processes of creativity, intuition, imagination, insight, and illumination"(p. 65). He states that something is lost between the process of problem solving and the product that we read. Often times, students will write solutions to problems in the way that they read them in textbooks (Shield & Galbraith, 1998; Liljedahl, 2006) or they way that they are presented through classroom instruction (Liljedahl, 2006).

Writing has also been linked to successful problem solving by supporting the development of students' metacognition. Garofalo and Lester (1985) designed a metacognitive framework consisting of four stages that students passed though while solving, which are 1) orientation, 2) organization, 3) execution, and 4) verification. Their work suggested that students who did not pass through all four stages possibly lacked in complete understanding of the problem. Pugalee (2004) investigated the impact of writing on metacognition by analyzing written samples of student work using the theoretical framework set forth by Garofalo and Lester to link writing in a mathematics classroom and metacognition.

Pugalee (2004) conducted a study involving twenty students that were taking an introductory high school Algebra course. The students were divided into two groups. One day Group 1 would solve a mathematical problem using a think aloud process while being videotaped as Group 2 would use a written process to solve a mathematical problem. The next day, the groups would rotate and use the other strategy. Pugalee (2004) found that "students who wrote about their problem solving processes produced 32 correct solutions out of the total 60 solutions, whereas the think aloud students produced 20 correct solutions" (p. 37). He found that the proportions were significantly different ($p < .05$). Students were thinking about the processes or methods of problem solving as they attempted a problem. Pugalee (2004) recognized the importance of writing for improved metacognition of students.

Other forms of writing in mathematics classrooms are worth discussing. Ntenza (2006) reported on a study that involved various types of writing. For example, one suggestion for a writing activity was to have students spend fifteen minutes writing about an entire unit. In the written assignment, students were expected to discuss the main goals of the unit, confusion of concepts, understanding of concepts, feelings about the unit, and suggestions. Ntenza (2006) was able to identify two main forms of writing, symbolic and mathematical. Symbolic writing was defined as writing involving symbols when completing traditional problems in class. Ntenza (2006) used a model set forth by previous researchers, Davison and Pearce (1990) to define mathematical writing. There were four types of mathematical writing that took place in Ntenza's study 1) direct use of the language, 2) linguistic translation, 3) summarizing and interpreting, and 4) creative use of language. Direct use of language is where students essentially copy examples and definitions from the board into their notebooks. Linguistic translation is a form of writing in which students interpret mathematical symbols by writing them with words. Summarizing and interpreting requires students to explain material involving the mathematical concepts they are using when solving problems. Creative use of language requires students to complete a project, and then investigate and explain mathematical information by writing an assignment. Ntenza (2006) noted that "there is very little mathematical writing taking place" involving explanations in his study (p. 337). A possible limitation to the research provided by Ntenza (2006) was the suggestion to give students fifteen minutes to write about the main goals of a unit or discuss confusing concepts. Allowing students fifteen minutes to write might not be enough time to elicit the expected results a teacher would hope to see. A second possible limitation with Ntenza's work could be the writing prompts that he used for his study. As noted previously, appropriate writing prompts are important (Shield & Galbraith, 1998) with a list provided by Pugalee (2005) that should lead to better writing by students. Teachers need to pay close attention to writing prompts if they want students to write in a way which will benefit the students. My study filled in the gaps of this research by employing good writing prompts provided by Pugalee (2005).

## Conclusion of Literature Review

Many researchers have studied problem solving and felt that more research regarding the topic is warranted (Lester, 1994; McLeod, 1989; Polya, 1962; Schoenfeld, 1985; Silver, 1985). Furthermore, the processes of writing and problem solving have been linked (Cai, 1994; Flower & Hayes, 1983; Pugalee, 2005; Pugalee, 2001) with writing showing a positive effect on students' problem-solving abilities.

Writing in mathematics classrooms will benefit students from mathematical perspectives, such as problem solving. Ediger (2006) advocates writing across the curriculum. When a mathematics teacher models and emphasizes good writing techniques for students, they will "learn to write as well as write to learn in mathematics" (p. 120). Writing requires students to reflect on learned concepts and provides teachers with an instrument to investigate what students truly understand (Baxter, Woodward, & Olson, 2005; Ediger, 2006).

The central framework of this dissertation is that writing can be used as an effective tool for students to solve problems and learn concepts in a statistics classroom. Various forms of writing have produced positive effects on learning mathematics (Ntenza, 2006; Hamdan, 2005; Johanning, 2000; Pugalee, 2005; Pugalee, 2004; Pugalee, 2001; Rose, 1990; Shield & Galbraith, 1998; Waywood, 1994). While this may be true, other studies have shown mixed results regarding the effects of writing in a mathematics classroom (Porter & Masingila, 2000, Ntenza, 2006), possibly due to the time given to students to write or to the type of writing prompts being used.

The purpose of this study was to further investigate the effects that writing has on learning mathematics for students enrolled in an Advanced Placement Statistics course. If writing is positively linked to the understanding of statistics in a high school classroom, it will enable teachers of Advanced Placement Statistics to use writing to help students be more successful. To add as enrichment of this study, a specific form of problem solving, problem posing (English, 1997; Silver, 1994), was investigated.

## Significance of this Study

Everything being discussed which examines writing and statistics addresses a review of literature showed the importance of writing in mathematics. Previous research discussed how the use of writing affects the understanding of mathematical concepts. Researchers claimed that different forms of writing have yielded positive effects on learning mathematics (Hamdan, 2005; Johanning, 2000; Pugalee, 2005; Pugalee, 2004; Pugalee, 2001; Rose, 1990; Shield &Galbraith, 1998). The purpose of this study was to further investigate the effects that writing has on learning statistics for students enrolled in an Advanced Placement Statistics course. My study intended to promote writing in a statistics course using writing prompts that will promote deeper levels of thinking regarding statistics. In an effort to have students become comfortable with their own writing, the instructor intended to start the students with a couple of easy problems. This was the main reason that the problems were rated based on difficulty. The populations of other studies have also been primarily different than from my study.

For instance, Cisero (2006) found that writing has a positive effect on student performance in class for lower-achieving students in college. My study involved higher-achieving students who were college bound. The higher-achieving students in this situation benefited from other options when learning, such as writing. My study contributed to a different population of students.

Problem-solving in the field of mathematics is important to determine if students are able to use concepts learned for application (English, 1997; Lesh & Harel, 2003; Schoenfeld, 1982; Silver, 1994). Research showed that problem solving in a mathematics classroom has been investigated for many years. This study intended to investigate problem-solving processes in statistics. Writing was used as a tool for learning that also enabled the researcher to study these problem-solving processes. This helped investigate if writing promoted the understanding of statistical concepts for students.

Finally, my study added to the field of mathematics education by also investigating problem posing. This particular problem posing required students to pose a problem when given a statistical concept, unlike Silver (1994) who studied how his students reformulated questions to answer before answering the primary question and

English (1997) who gave her students either a number only or verbal statement only to pose problems.

# CHAPTER 3: METHODOLOGY

This study sought to answer two primary questions involving writing in an Advanced Placement Statistics classroom. Students' problem-solving processes were investigated through the practice of writing as they solved problems in a high school Advanced Placement Statistics course. This research study examined the effects of the use of writing on students' understanding of statistical concepts. More specifically, the research questions are

1. How do students' problem-solving processes through writing provide a rich description as they solve problems in a high school Advanced Placement Statistics course?
    - How can one describe the problem-solving ability in students' writings as they solve problems in a statistics course?
    - How can one describe the conceptual understanding in students' writings as they solve problems in a statistics course?
    - How can one describe the procedural understanding in students' writings as they solve problems in a statistics course?
    - How can one describe the mathematical content in students' writings as they solve problems in a statistics course?
    - How can one describe the mathematical reasoning in students' writings as they solve problems in a statistics course?

- How do the students' writings improve over time?
- How can one describe problem posing by students as they solve problems in a statistics course?

2. Are there any differences between the problem-solving processes of problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning when analyzing written samples of students using ratings from a rubric?

## Participants

There were 14 participants who were enrolled in an Advanced Placement Statistics course. The participants were eleventh- and twelfth-graders who attended East Henderson High School, located in East Flat Rock, North Carolina. The city of Flat Rock is located in Henderson County. The county has a population of approximately 90,000 people and is home to four high schools. The student population of East Henderson High School was 1079 and composed of Whites (88%), Hispanics (11%), and African-American (less than 1%). The socioeconomic status of the students for East Henderson High School was predominantly middle and lower class. Approximately 34% of students at this school receive free or reduced lunch. In this AP Statistics classroom, there were 9 (64%) female students and 5 (36%) male students, 13 (93%) White students, and 1 (7%) multi-racial student. No students in this AP Statistics class received free or reduced lunch.

Of the participants, there were seven students identified as academically gifted in mathematics. East Henderson High School adheres to the guidelines set forth by Henderson County, approved by North Carolina's Department of Public Instruction, when identifying students as academically and intellectually gifted. Henderson County has an academically and intellectually gifted staff in place that uses a variety of indicators when identifying students. These sources include an intelligence quotient, standardized achievement tests, teacher recommendations, and individual nominations. Parents, students, or members of the community may make nominations for students. In high school, students who score greater than or equal to the 92$^{nd}$ percentile on the eighth-grade end-of-grade exam, end-of-course exams, Preliminary Scholastic Aptitude Test (PSAT), Scholastic Aptitude Test (SAT), or American College Test (ACT) are considered for identification. Other considerations for identification include students who rank in the top ten percent of their class or students who score a five on an Advanced Placement examination.

The writing levels of students were determined prior to the study by using the North Carolina writing scores that students earned in the tenth grade. The primary reason for considering writing levels was to confirm that students were proficient at writing since the study involved large amounts of writing. A student is classified as proficient in writing ability if he scores a Level III or Level IV on the North Carolina General Writing Assessment in the tenth grade.

According to the North Carolina Department of Public Instruction (2008), to score a Level III, students are expected to "consistently demonstrate mastery of grade level subject matter and skills and are well prepared for the next grade level. Students performing at Achievement Level III maintain consistent control of the purpose, audience, and context of the response. A sense of organization, a logical progression of ideas, and sufficiently developed support and elaboration are present. Students display a consistent control of conventions and style and are well prepared for the next grade level" (p. 1). Students who score a Level IV on the writing exam are expected to "demonstrate the use of higher order thinking skills in presenting a unified progression of ideas while examining the relationships between and among those ideas. In-depth support and elaboration is shown through the use of precise, appropriate language. Students display a skillful use of conventions and style clearly beyond that required to be proficient at grade level work" (p. 1). All students in this study scored at level 3 or above.

## Design of Study

A mixed-methods research design was implemented. This approach enabled the researcher to better manage the complex nature of narrative data of problem-solving behaviors exhibited by students with a qualitative approach, while permitting the researcher to check for statistically, significant differences between the problem-solving facets with a quantitative approach. Since the two primary research questions are interconnected, a mixed-methods approach is warranted and preferred (Tashakkori & Creswell, 2007).

The quantitative piece of this study involved using a rubric, described later in this chapter, to code student writing samples by assigning numerical values. The values were analyzed using software to check for statistically, significant differences between the five categories on the rubric. The qualitative piece of this study employed a method of coding to identify the five facets of problem solving: problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning (Pugalee, 2005). The identification enabled the researcher to look for commonalities and differences for students' writing samples that were coded the same.

The qualitative component also included discussion of subgoals and problem posing that provided more enriching findings and assisted answering the qualitative research question.

The researcher was the instructor for this class. Wong (1995) asserts that conflicts may possibly arise when the researcher is also the instructor. When a person assumes both roles, he might be torn between helping a student while possibly influencing the data being collected and not helping the student. The researcher does not want to influence the data while the teacher is professionally obligated to assist students. This type of conflict did not occur during this study.

The instructor had ten years of teaching experience, instructing primarily Advanced Placement Statistics, geometry, discrete mathematics, and Algebra 2 courses. The instructor has a Bachelor's degree in mathematics and a Master's degree in mathematics education. The instructor taught an Applied Statistics course at a four-year university and Advanced Placement Statistics for four years at East Henderson High School prior to the study. He is also a National Board Certified Teacher in mathematics for Adolescence and Young Adulthood.

The instructor taught the only section of Advanced Placement Statistics offered at the school. The intervention consisted of an eight-week unit that involved new concepts and concepts that have already been learned. The material in the unit included histograms, boxplots, standard normal curve, normal distributions, z-scores, probability, sample design, binomial distributions, geometric distributions, t-tests, and chi-square tests.

Before the intervention, a typical day in this class involved students learning in a primarily traditional approach. At the beginning of class, homework problems from the previous day's lecture were reviewed. The instructor then answered specific questions. The topic of the day followed, usually in lecture format. Technology, primarily a TI-84 Plus graphing calculator, was utilized by the instructor when appropriate. With roughly fifteen minutes remaining in class, an assignment was given to students. This allowed students to start the assignment and see if any questions arose before they left class. When answering questions in statistics, some writing already took place. For example, a student might write an explanation when comparing which batter in the sport of baseball from different eras is better. Students were expected to use standard deviations in their argument.

This researcher wanted to investigate writing in statistics to examine if the process of writing deepens the understanding of statistical concepts for students. Writing prompts were designed to get students to reflect on the task of solving the problem and to get students to better understand statistical concepts. The writing prompts are based on fifty activities for writing in mathematics offered by Pugalee (2005). See Appendices A-X for all of the problems using writing prompts for this study.

During the intervention, the classroom practices were different than before the intervention. The following section describes the intervention and how a day in the classroom during the intervention looked.

## Intervention

The study lasted eight weeks and spanned sixty-four days. Since East Henderson High School employs the block schedule, the length of each class period is ninety minutes. There were two goals of the writing that took place in the study. The first goal was to help students understand statistical concepts being learned as they solved problems. The second goal was to help the researcher investigate if writing gives teachers a better understanding of students' problem-solving processes.

Students described their thinking through writing as they solved problems three times a week. There were twenty-four different problems used in the study. On the days that students did not complete writing assignments, the instructor reviewed writing which took place the day before and allowed students the opportunity to discuss their writing. Some days students discussed their writing with the whole class; other days they did so in small groups. Also, on days which students did not complete writing assignments as they solved problems, students wrote collaboratively. The goal was to have students writing every day because writing should be done often in a mathematics classroom (Pugalee, 2005). On the days that students completed problems, a review of the previous day's assignment occurred during the first forty-five minutes of class. After the review, the students were given approximately forty-five minutes of class time to complete the problem of the day, which involved writing.

At the beginning of the study, the instructor discussed expectations regarding problems, modeled examples (Pugalee, 2005), and discussed aspects of a safe, classroom environment to exchange ideas (Pugalee, 2005; Yackel, 2000). The instructor gave a practice problem for students to complete as a homework assignment. The instructor completed the problem in written form, photocopied it for each student, and discussed the solution and certain writing practices that the students might find beneficial. Once the intervention started, the problems were collected after each session so that the instructor could provide feedback for the students. Feedback was given frequently to increase learning (Black & William, 1998). The feedback included information on whether the student got the problem correct and clarity of explanations as indicated below. Some aspects of writing, such as grammar, were not part of the feedback for students. Pugalee (2005) states that it is “imperative that teachers remember that the goal of writing in mathematics is to support students’ understanding of mathematical ideas and concepts” (p. 116). Student responses were also photocopied for analysis and analyzed using rubrics. The rubrics are discussed in the section titled Measures.

## Data Collection

A total of twenty-four problems were given throughout the course of the eight-week unit. Students completed three different problems each week, on Mondays, Wednesdays, and Fridays. All of the completed work was kept in folders, provided to the students by the instructor. The students were instructed that they should take the responsibility of writing seriously, and that the assignments counted toward their final average for the course: each problem counted as a quiz grade. The researcher scored each problem using a rubric designed by Pugalee (2005) regarding five areas: problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning. Each quiz grade was determined by using the sum of the scores for all of the categories and dividing by 20. For example, a student received a score of 4 for problem-solving ability, a score of 3 for conceptual understanding, a score of 4 for procedural understanding, a score of 2 for mathematical content, and a score of 3 for mathematical reasoning. The sum of these scores equals 16. Thus, 16 divided by 20 equates to 80 percent for the quiz grade.

The twenty-four problems involved various statistical concepts. The problems came from a variety of sources, including the textbook currently being used for the course, other statistical textbooks, released exams from The College Board, and other supplementary materials. By using many different sources to find and create problems, this allowed the researcher the opportunity to choose problems that represented different topics in statistics. Writing prompts were added to the problems by the researcher to help enrich the writing experience as the problems were completed by the participants.

## Panel of Educators

A panel of four educators, which included the researcher, was assembled for the study. The other three educators worked in the same school district at East Henderson High School with the instructor. One educator had ten years of teaching experience and had taught Advanced Placement Calculus for four years. A second educator had thirty-six years of teaching experience, which included teaching Advanced Placement Calculus and Advanced Placement Statistics. A third educator taught Advanced Placement Statistics for six years at a school in Henderson County and had thirty-three years of teaching experience. After problems with writing prompts were created, the panel rated the problems by level of difficulty before they were given to students to complete. After the students completed the writing prompts, a random sample of de-identified student responses was given to two panel members to score for five problem-solving facets using a rubric provided by the researcher and compared to the scores of the researcher to check for inter-coder reliability.

The panel of four mathematics teachers at the secondary level individually classified problems using three levels of difficulty: easy, medium, or hard. Pugalee

(2004) uses a similar method when studying problem solving and metacognition. By classifying the problems based on difficulty, it allowed the instructor to start with easier problems for the first couple of writing assignments. This enabled the participants the chance to gain a comfort level with writing in statistics in a different way than they are used too. See Appendices A-X for all of the questions used in the study.

## Measures

Problem solving often requires students to use multiple strategies, including the identification of subgoals (Polya, 1962). When students write down subgoals or certain steps to a problem to help them solve it, the activity of writing is taking place. Other forms of problem solving, such as problem posing, have become of interests to researchers over the past 20 years. The identification of subgoals and problem posing are two problem-solving strategies that will be investigated when analyzing the data for this study.

A first level of analysis involved looking for problem-solving processes, specifically problem posing and the use of subgoals (Polya, 1962). Silver (1994) argues that problem posing helps students solve problems. One form of problem posing involves students writing problems of a different context involving mathematical concepts being learned. For this study, writing prompts were designed to require students to write problems using statistical concepts. Students were also expected to describe the process that they employed when solving problems. This enabled the researcher to better identify and analyze subgoals that students used.

A second level of analysis involved applying a rubric to assess students' problem solving processes in the facets of mathematical content, conceptual understanding, procedural understanding, problem-solving ability, and mathematical reasoning. For each problem, performance summaries for the class included means and standard deviations. Representative responses from the students' written narratives provided a characterization of thinking relative to each of the five facets. This allowed for viewing the overall performance of the students within and across the twenty-four problems. Hierarchical Linear Modeling (HLM) will be utilized with statistical software to check for statistical differences. The data and results will be displayed in a table noting significant differences.

Hierarchical Linear Modeling (HLM) is a statistical method that analyzes data in a nested structure. Bryk and Raudenbush (1992) state, "Educational research is often especially challenging because studies of student growth often involve a doubly nested structure of repeated observations within individuals, who are in turn nested within organizational settings" (p. 2). For this study, the questions that each student solved were nested within the students.

While some other conventional statistical methods were considered for this study, it was believed these methods would lead to possible violations of assumptions. According to the National Assessment of Educational Progress (2006), students who attend the same school "share many common, educationally relevant experiences that affect academic performance" (p. 4). Knowing this fact, the author argued that a violation of independence would occur, in turn leading to unacceptable levels of bias. HLM was used for this study and consisted of two levels. Different variances were assumed for each level.

This two-level analysis provided a characterization of students' problem-solving processes evident in their written descriptions as they completed statistics problems. Writing samples were analyzed by the researcher and the panel of four educators to identify problem-solving techniques of student thinking regarding statistical concepts, which include histograms, boxplots, standard normal curve, normal distributions, z-scores, probability, sample design. The analysis also looked for problem-solving processes that students incorporated when solving problems in statistics. The problem-solving processes included the understanding of the problem, strategies and reasoning, and problem posing. Each writing sample was analyzed to see if the student had an understanding of statistical concepts for the problem. Each writing sample was coded to investigate if the student reasoned correctly and chose an appropriate strategy to solve the problem. Each writing sample was also analyzed to search for conceptual understanding.

## Rubric for Problem-Solving Processes

Pugalee (2005) designed and used a rubric to measure five problem-solving processes: problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning. The same rubric was used for this study. See Appendices Y and Z for this coding system.

To investigate a student's problem-solving ability as he completed a problem, the researcher and panel of experts looked for evidence that the participant identified the goal of the problem or task, developed a plan that shows an understanding of all components of the problem, and the plan was executed with no errors.

To investigate a student's conceptual understanding as he completed a problem, the researcher and panel of experts looked for evidence that the participant identified and provided information about major concepts, and supplied examples or illustrations with explanations when appropriate.

To investigate a student's procedural understanding as he completed a problem, the researcher and panel of experts looked for evidence that the participant selected and executed appropriate strategies, and whether the representations and algorithms were appropriate.

To investigate a student's procedural understanding as he completed a problem, the researcher and panel of experts looked for evidence that the mathematics were accurate, all mathematical concepts and ideas were accurately identified, and mathematical terms were used appropriately.

To investigate a student's procedural understanding as he completed a problem, the researcher and panel of experts looked for evidence that the participant completely and accurately provided justification for major steps or processes, and defended the reasonableness of the answer with supporting reasons.

## Validity and Reliability

Validity was addressed prior to the study being conducted. The researcher chose and designed appropriate questions that measured the concepts being covered. The panel of experts was informed of the concepts being covered in the study and asked to make suggestions if they felt a question was not relevant. Validity of the questions using writing prompts was not an issue since a panel of four educators in the field of mathematics and statistics read the problems and verified that the content of the problems represents the content of Advanced Placement Statistics curriculum. The members of the panel accomplished this task by reviewing the North Carolina Standard Course of Study for Advanced Placement Statistics. The members then read the problems and verified that the content corresponded to the curriculum.

Internal validity was addressed by using the writing scores of students on the North Carolina General Writing Assessment. Since all of the students who participated in this study scored a Level III or Level IV, the writing ability of students was likely not a confounding variable that contaminated the conclusions that were drawn by the researcher.

Inter-coder reliability was also addressed before the study. The researcher used a designed rubric by Pugalee (2005) for the panel of educators to use when scoring the writing prompts. Prior to the panel scoring student responses, the researcher met with each panel member individually and explained what to look for and how to use the rubric when coding. The researcher also modeled a problem and its coding for the panel. Afterwards, each panel member received a sample response that was indicative of a student response. They coded the response. This process allowed the researcher to determine if each member was applying the rubric appropriately.

Once the writing samples were collected, the researcher scored all of the student responses. Two members of the panel were given a random sample of seventy (21%) student responses to code. The coded responses of the panel members were compared to the scores of the researcher check for inter-coder reliability, looking for a minimum of 85% exact agreement and 100% adjacent agreement. To measure the inter-coder reliability, an agreement rate (AR) was used. According to Orwin (1994), it is also called the percent agreement index. The AR equals the number of observations agreed divided by the total number of observations.

One member of the panel had an exact agreement rate of 91% and an adjacent agreement rate of 100% with the researcher. The second member of the panel had an exact agreement rate of 85% and an adjacent agreement rate of 99% with the researcher. The overall exact agreement was 88%, and the overall adjacent agreement was 100%.

## Ethics and Compliance

Before the study began, the principal of East Henderson High School reviewed letters of consent and assent written by the researcher. Students received a letter of consent to take to parents and a letter of assent describing the nature of the study for students to sign. The letter assured the parents that no risk is involved, and that the research had full IRB approval from The University of North Carolina at Charlotte.

The study followed the ethical guidelines described by the American Educational Research Association (2000), which included honesty, integrity of research, and that participants could withdraw from the study at any time. No ethical issues arose during the study. Parents were reassured that the practice of the study was ethical, and that the students received proper instruction, following the guidelines set forth in the North Carolina Standard Course of Study.

# CHAPTER 4: QUALITATIVE DATA ANALYSIS

This chapter discusses the findings of the qualitative component of the study. The discussion of the qualitative component will rely upon narrative examples to illustrate problem-solving processes. To illustrate how students' writings make visible problem-solving processes, the discussion will involve looking at a student's writing sample for each score using a rubric created by Pugalee (2005) on the five problem-solving processes: problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning. The research sub-questions will be utilized to help guide the reader for clarity of the findings. After looking at writing samples for each score, commonalities and differences will be noted in a conclusion section following each sub-question for sub-questions one through six.

## Research Question

How do students' problem-solving processes through writing provide a rich description as they solve problems in a high school Advanced Placement Statistics course? To explore this research question, seven sub-questions were developed.

### Sub-question #1

How can one describe the problem-solving ability in students' writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score using a rubric created by Pugalee (2005). For Writing Prompt #20, see Appendix

T, students were asked to analyze a statement that over half of Indiana corn producers did not get back from their corn crop the money they put into seed, fertilizer, etc. The problem stated that a random sample of 800 farms had been chosen. After a brief audit on each of these farms, it was discovered that 405 farms did not recover their costs.

Using a rubric designed by Pugalee (2005), a student scored a four for the problem-solving process of problem-solving ability if the student identified the goal of the problem or task, developed a plan that showed an understanding of all components of the problem, and executed the plan with no errors. A student scored a four for problem-solving ability as she solved this problem. She wrote:

> I arrived at this conclusion by using a 1-proportion z-test. This test was appropriate because we were interested in the proportion of Indiana corn producers who did not make profit from one sample. I stated the hypotheses testing the proportion .5 and verified the conditions necessary to perform the test. I chose .05 for my significance level since it was not given. I calculated the z statistic to be .354 and by using this standardized value, I found the p-value to be .368. Since this number is greater than our alpha of .05, therefore we failed to reject $H_0$, concluding that the actual proportion is less than or equal to .5. It does not appear that the proportion of corn producers who did not make a profit is more than .5.

She immediately started the four-step process of a significance test. She identified the population of interest and what was being investigated. She checked the appropriate assumptions to make sure that there were no violations and found the correct test statistic. She calculated the correct p-value and wrote a correct conclusion. She chose to describe how she solved the problem after she had completed the problem.

Using a rubric designed by Pugalee (2005), a student scored a three for the problem-solving process of problem-solving ability if the student identified the goal of the problem or task and developed a plan that showed an understanding of the problem but contained minor errors in executing the plan. A student scored a three for problem-solving ability as she solved this problem. She wrote:

> At the 5% significance level with a p-value of .638 > .05, we reject $H_0$. Therefore, it appears that over half of Indiana corn producers did not get back from their corn crop the money they spent on their crops.

She identified the appropriate significance test, population of interest, and what she investigated. She calculated the correct test statistic but not the correct p-value. Since students obtained the p-value using a graphing calculator, there was no work to analyze to see where her mistake occurred. The researcher wrote a note, at this point, inquiring how the mistake happened. When the papers were returned the next day in class, she went to the researcher and explained that she had drawn a lower-tail test picture on a different sheet of paper and showed that to the researcher. This made sense since her p-value equaled .638 when the correct p-value equaled .362, which is .638 subtracted from 1. This mistake occurred because an upper-tail test should have been drawn, which would have corresponded with her hypotheses. By calculating the incorrect p-value, it led her to draw the opposite conclusion that she should have drawn. If she had drawn the correct picture, she would have calculated the correct p-value and made the appropriate conclusion.

Using a rubric designed by Pugalee (2005), a student scored a two for the problem-solving process of problem-solving ability if the student identified the goal of the problem or task but misinterprets one or more of the components of the problem, and the plan indicated minimal understanding of problem. A student who scored a two for problem-solving ability as she solved this problem correctly identified the significance test and hypotheses for the problem. She failed to mention the population of interest and what was being investigated within the context of the problem. While she identified the correct significance test, she wrongly identified it as a p-statistic, instead of a z-statistic. She incorrectly found the test statistic to equal .36, instead of .35. She also used the test statistic as the p-value. She wrote:

> Let $\alpha = .05$. At the 5% significance level with a p-value = .36 > .05, we fail to reject $H_0$. Thus, it appears that more than half of the Indiana farmers did not get back the money that they spent on their corn crops.

In writing, she demonstrated that she knew to use a one-sample z significance test to solve this problem but executed her plan poorly. Interestingly, her test statistic that was incorrectly calculated actually coincided with the correct p-value. It was unclear from her work if this was a true coincidence.

Using a rubric designed by Pugalee (2005), a student scored a one for the problem-solving process of problem-solving ability if the student did not identify the goal of the problem or task but the response showed some evidence of understanding the general nature of the problem, and the student did not develop a plan. For Writing Prompt #6, see Appendix F, students completed a problem involving foresters and lumber harvested from various tree species. They were given data involving chest height of trees and yield in board feet. Students were asked to construct an appropriate model for the given data and then comment on the quality of the model. A student scored a one for the problem-solving process of problem-solving ability as she solved this problem. She wrote, “This model shows how the data is now roughly linear.”

This one sentence was her complete response to the question. She had two columns of data written on her paper showing that she had calculated the logarithms for both variables of the problem. She did not identify the primary statistical concepts needed to answer the problem. She also failed to develop any type of plan, but seemed to think that she needed to straighten the data by using the logarithm function. Students are taught to straighten data for regression purposes, if necessary. Straightening data is a phrase used to describe the process of taking curved data and making it approximately linear. This can be done by finding the logarithms of the response variable values. While the data for this problem did not need to be straightened, this act alone was enough to convince the researcher that she knew that regression was needed. A more complete response would have involved a scatter plot of the given data, a least-squares regression line, a correlation coefficient, a coefficient of determination, a residual plot, and an explanation regarding the residual plot.

Using a rubric designed by Pugalee (2005), a student scored a zero for the problem-solving process of problem-solving ability if the student showed no evidence of understanding the goal of the task or problem, and made no attempt to specify or develop a plan. No study participants scored a zero for the problem-solving process of problem-solving ability.

Conclusion for Sub-question #1

Of the 329 writing samples collected for the study, 76.3% of the samples were coded a four for problem-solving ability. Student writing samples that were coded a four generally had the same characteristics. The responses were clear and detailed when identifying the goal of a problem. Students whose responses were coded a four unambiguously described their methods of solving the problem. A final commonality of these writing samples was the structure of the responses with the majority of these responses written in paragraph form, as evidenced by a student in the previously discussed example. While she was able to clearly detail and describe her method of solving the problem.

Of the 329 writing samples collected for the study, 13.7% of the samples were coded a three for problem-solving ability. These writing samples were similar to the writing samples that were coded a four in that they were usually written in paragraph form, students would make minor errors, such as drawing incorrect pictures for significance tests that would in turn lead a student to the wrong p-value. The minor errors committed were the primary reason that student responses were coded a three, instead of a four.

Of the 329 writing samples collected for the study, 6.7% of the samples were coded a two for problem-solving ability. Students whose work was coded a two seemed to recognize the primary statistical concept and approach that was needed to solve the problem but lacked the ability to correctly carry out the plan. This was evidenced by the numerous errors made by the student discussed previously whose written response scored a two as she attempted to solve the problem.

Of the 329 writing samples collected for the study, 3.3% of the samples were coded a one for problem-solving ability. The primary difference for a writing sample to be coded a one instead of a two was that the student did not correctly identify the appropriate plan to take when solving a problem. There was limited writing to demonstrate analysis. While there was little work exhibited by students, there was enough work to support the conclusion that students had a vague idea of the nature of the problem. This was demonstrated when the student discussed previously whose written response scored a one made a one-sentence response regarding data being roughly linear.

Sub-question #2

How can one describe the conceptual understanding in students' writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score using a rubric designed by Pugalee (2005). For Writing Prompt #1, see Appendix A, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, *A* or *B*, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable.

Using a rubric designed by Pugalee (2005), a student scored a four for the problem-solving process of conceptual understanding if the student identified and provided information about major concepts, and supplied examples or illustrations with explanations when appropriate. A student scored a four for conceptual understanding as he solved this problem. He calculated and identified the cost of Machine A as $11,800 and the cost of machine B as $11,010. He calculated and wrote:

> $E(X) = \mu_x = 0(.5)+1(.25)+2(.15)+3(.1) = .85$ repairs per year. .85 * 3 = 2.55 repairs * $200 = $510. 10,000 + 510 = $11,010. I would recommend buying Machine B because over the three years that the machine will be in operation, it will be cheaper. The expected number of repairs over the next three years is 2.55. Therefore, with this number of repairs, Machine B will cost $11,010, and machine A will cost $11,800. So, it is a better idea to purchase Machine B.

Notice how he demonstrated through his calculations how to properly find the expected value, even using the appropriate symbols for statistics. He then correctly applied the statistical concept to the context of the problem using the phrase "expected number of repairs." He identified and provided information about the major concept of this problem and illustrated his understanding using the correct formula when finding an expected value.

Using a rubric designed by Pugalee (2005), a student scored a three for conceptual understanding if the student identified and provided information about major concepts while possibly omitting minor details. The student may also have used examples or illustrations when appropriate but failed to effectively relate them to mathematical concepts. A student scored a three for the problem-solving process of conceptual understanding as she solved this problem. She calculated and identified the cost of Machine A as $11,800 and the cost of machine B as $11,010. Isabella wrote:

> I would recommend buying machine B because the total cost for Machine A is $11,800. This is the base price plus the flat repair contract for three years. Machine B's total expected cost is $11,010. This is the base price plus the expected cost of repair based on a plan that charges per repair. I found this by calculating probability. 10,500 + 200(3(0*.5+1*.25+2*.15+3*.1)).

From her calculations, it is clear that she understood the primary statistical concept being used in this problem, but omitted a minor detail regarding expected value. She used the phrase "expected cost." While her calculations demonstrated the process for finding the expected value, she did not discuss the expected number of repairs and how she applied that to the cost per repair. She indicated that she found the expected cost by calculating the probability. This was vague and did not completely convince the reader that she understood the statistical concept completely. By having students write in statistics, it enables a teacher to better grasp the conceptual understanding for students of topics being learned.

Using a rubric designed by Pugalee (2005), a student scored a two for the problem-solving process of conceptual understanding if the student identified and provided support for major concepts but may have had minor errors in logic or understanding, and minor details were ignored or supported with incorrect or flawed thinking. A student scored a two for conceptual understanding as she solved this problem. She identified the cost for Machine A as $10,600 and the cost of Machine B as $11,100. She notated, "$E(x) = .85$." This notation indicated that she knew the primary concept that needed to be used to solve the problem. Yet, she made errors trying to apply it to the problem. She wrote:

> I would suggest that Machine B be purchased. For one year, with no repairs, Machine B is $100 cheaper than Machine A. There is a .5 probability that Machine B will not need any repairs, while one repair has a probability of only .25. Machine B would save more money.

When analyzing her written explanation, it is clear that she chose to go away from the primary concept of expected value since she did not use the concept when drawing a conclusion about which machine to choose.

Using a rubric designed by Pugalee (2005), a student scored a one for the problem-solving process of conceptual understanding if the student did not correctly identify major concepts and the information contained errors in logic or understanding. There were no scores of one for the example involving expected value. To analyze a written sample submitted by a student that received a score of one, we will look at a different problem that involved probability and rules of probability. For Writing Prompt #8, see Appendix H, students were faced with a problem regarding blood disease and the accuracy of a blood test correctly identifying if people actually have the disease. The primary concepts involved in this problem included conditional probability, the complement rule, and basic rules of probability. The basic rules of probability include the fact that the sum of the probabilities of all of the possibilities that compose the sample space for a certain event is equal to one and that a probability of a possibility in the sample space is between zero and one, inclusively.

A student scored a one for the problem-solving process of conceptual understanding as he solved this problem. Students were asked to describe the rules of probability used when solving the problems and the importance of each rules. He wrote:

> The 2% given in this problem is the $\hat{p}$ of persons who have the disease. When the $\hat{p}$ is 2%, 96% is the power (of persons with the disease). This would be in a test where $H_0 : p = 0$ and $H_0 : p > 0$, dealing with the proportion of persons with the disease. For a test where 98% of the P(probability of persons without the disease), 94% is the power. This would be in a test where $H_0 : p = 0$ and $H_0 : p > 0$, dealing with the proportion of persons without the disease.

It is evident that he did not know the major concepts involved with the problem. He described a significance test involving proportions while his actual work to solve different aspects of this problem did not include a significance test. He also mentioned power, which had nothing to do with this problem. The biggest difficulty that he faced, when trying to solve this problem, appeared to be a lack of knowledge regarding which statistical principles to use. When trying to solve a problem, it is essential for students to understand the primary concepts of a problem in order to develop and execute a strategy that will yield the solution.

Using a rubric designed by Pugalee (2005), a student scored a zero for the problem-solving process of conceptual understanding if the student made no attempts to identify or provide information about major concepts or the information had no mathematical soundness. No study participants scored a zero for conceptual understanding.

Conclusion for Sub-question #2

Of the 329 writing samples collected for the study, 57.8% of the samples were coded a four for conceptual understanding. Students whose writing samples were coded a four demonstrated that they knew the primary statistical concept involved in the problem that they were solving. They were able to clearly articulate the relationship of the statistical concept and the context of problems the participants solved. This was evidenced by a student for an example discussed previously when he correctly identified the statistical concept of expected value and the correct context of expected number of repairs for the machines.

Of the 329 writing samples collected for the study, 23.7% of the samples were coded a three for conceptual understanding. There was a subtle difference in student samples that were coded three, instead of four. Students whose work was coded a three knew the statistical concept related to the problem. However, they made minor mistakes when drawing conclusions in the context of the problem. Referring to a previous example of a student response written by a student, it was clear that she knew the primary concept of expected value and found the correct mathematical solutions but failed to contextualize the results in a completely accurate manner. Her mistake was subtle but nonetheless a failure to communicate the expected number of repairs.

Of the 329 writing samples collected for the study, 13.7% of the samples were coded a two for conceptual understanding. Students whose work was coded a two struggled to recognize the primary concept involved in solving a problem. While their work often contained the primary concept at some point during the process of solving the problem, it was never the focal point. Students never used the concept when finalizing answers and drawing conclusions. This can be seen when a student in a previously discussed example used notation that represents the statistical concept of expected value, but she never incorporated it into her final response when she drew conclusions.

Of the 329 writing samples collected for the study, 4.9% of the samples were coded a one for conceptual understanding. Students whose writing samples were coded a one made an attempt to solve the problem while not being able to correctly recognize any of the relevant concepts. The writing samples showed no evidence of student understanding of applicable concepts, as shown in the example regarding a student who was unable to demonstrate the primary statistical concept of probability in his response. While the differences in coding student responses a three or four for students is subtle, the differences in student work that was coded one or two were more obvious since students at minimum had to recognize pertinent concepts related to the problem.

Sub-question #3

How can one describe the procedural understanding in students' writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score. For Writing Prompt #19, see Appendix S, students encountered a problem that large universities face every year regarding housing of students. A particular university provided housing for 10 percent of its graduate students. A housing official conjectured that more than 10 percent of graduate students were looking for on-campus housing and completed a survey of a random sample of 481 graduate students. Of those students, 62 responded that they were looking for on-campus housing. The students completing this problem were asked to decide if this was evidence that the university needed to increase housing for its graduate students. A second part of this problem informed the reader that 19 students of the 481 students did not respond. Students were then asked if this would change their recommendation to the university regarding on-campus housing for graduate students.

Using a rubric designed by Pugalee (2005), a student scored a four for the problem-solving process of procedural understanding if the student selected and executed appropriate strategies, and the representations and algorithms were appropriate. A student scored a four for procedural understanding as she solved this problem. She identified the correct procedure, a one-proportion z test. She checked the assumptions for a one-proportion z test to verify that they were met. She proceeded to complete a four-step process with no flaws. Step three of this process involved identifying the z statistic, $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$, substituting in the appropriate values for the problem, and calculating the z statistic. This further supports that the student selected the correct procedure and used appropriate algorithms.

There were no students that scored a three or two for the problem-solving process of procedural understanding for this problem. Using a rubric designed by Pugalee (2005), a student scored a one for the problem-solving process of procedural understanding if the student selected an inappropriate approach or selected the appropriate approach but could not begin implementation. Also, the representations and algorithms were not appropriate for the task. A student scored a one for procedural understanding as she solved this problem. She correctly identified the hypotheses and $\hat{p}$ for the problem. It was unclear where one step ended and the next step began with her work. This may be due to her lack of understanding regarding this problem. She identified what appeared to be a test statistic, $p = \dfrac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}$. She has the correct formula for the appropriate z statistic, yet she labeled it as a p statistic which does not exists. Since the formula for the statistic was correct, she was able to calculate the test statistic correctly. After calculating the test statistic, she listed the level of significance of .05. Her choice of level of significance is reasonable. Students are taught to decide a level of significance before they calculate a test statistic. Since she chose her level of significance after calculating the test statistic, this further supports her lack of procedural understanding for this question.

Using a rubric designed by Pugalee (2005), a student scored a three for the problem-solving process of procedural understanding if the student selected and executed appropriate strategies, and some representations and algorithms had minor errors but did not affect the solution. For Writing Prompt #16, see Appendix P, students were faced with a problem involving chronic kidney failure and mean phosphorous levels. Faced with data for one patient, students were asked to decide if there was strong evidence that the patient's mean phosphorous level was too high. Students were also asked to describe Type I and Type II errors in this case. Students were also asked to describe their steps when solving the problem.

A student scored a three for the problem-solving process of procedural understanding as she solved this problem. She wrote her hypotheses correctly, identified the appropriate test to conduct, computed the test statistic with no problems, and wrote a reasonable conclusion. She wrote:

> To test the hypothesis, I used the formula to find the one-sample z statistic: $z = \frac{\bar{x} - \mu}{\sigma / n}$. To find the $\bar{x}$ of my data, I simply found the mean of all 6 numbers which gave me 5.4. From there, I plugged in the other numbers to give me a z-score if 1.63.

She described the procedure that she took as she calculated the sample mean and then ascertained the correct z-score. However, she missed some important procedures during her process of solving the problem. She did not discuss the population of interest and specifically state what she was investigating in step one. In step two, the student identified the correct test to conduct but failed to check the assumptions to see if they were met. This is an important procedural step that she bypassed.

Using a rubric designed by Pugalee (2005), a student scored a two for the problem-solving process of procedural understanding if the student selected an appropriate approach, but the execution was flawed. Also, representations and algorithms were appropriate for the task but were not executed properly. For Writing Prompt #1, see Appendix A, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, *A* or *B*, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable.

A student scored a two for the problem-solving process of procedural understanding as she solved this problem. She wrote:

> (0)(.5) + (1)(.25) + (2)(.15) + (3)(.1) = .85. Since this problem involves finding the mean of a discrete random variable, I used the formula $\mu_x = x_1 p_1 + x_2 p_2 + ...$ to calculate the expected value of the repairs that machine B will amass over a 1 year period. With machine A, the department supervisor would have to pay $10,600 a year after purchasing the repair contract. Although machine B is more

> expensive initially, there is no repair contract. It is $200 for each repair. According to the chart, there is an expected value of .85 repairs per year. This indicates that there is a fairly strong probability that no repairs will be necessary. Therefore, I would recommend machine B.

She understood that the primary statistical concept of this problem involved expected value. While she chose the correct path to solve the problem, her execution was flawed. After finding the expected value, she recommended Machine B. She incorrectly reasons that there is strong probability that no repairs will be needed. When she looked at the aspects of costs, she mentioned the costs for Machine A after the first year. She failed to apply the concept of expected value to the context of this problem. Her lack of conceptual understanding seemed to affect her procedural understanding. She failed to mention anything regarding a three-year plan for each machine. She did not correctly compute three-year totals regarding each machine.

Using a rubric designed by Pugalee (2005), a student scored a zero for the problem-solving process of procedural understanding if the student showed no evidence of representations or algorithms that would have indicated an acceptable approach. No one that participated in the study scored a zero for the problem-solving process of conceptual understanding.

Conclusion for Sub-question #3

Of the 329 writing samples collected for the study, 79.3% of the samples were coded a four for procedural understanding. Student work that was representative of this code involved the student selecting the appropriate procedure and being able to carry out the procedure with no mistakes to arrive at the correct solution. Written responses receiving a code of four showed the careful attention to detail taken by students as they solved problems. Students made sure to notate in words or mathematical symbols every step of the procedure, which was evidenced in the student's response regarding the one-proportion z-test as she made sure to complete each step correctly and with no flaws. Students whose responses were coded a three did not provide evidence of all of the details involved in a problem while still being able to reach the correct conclusion. It was easy to recognize this difference when reading written responses.

Of the 329 writing samples collected for the study, 6.7% of the samples were coded a three for procedural understanding. Student samples that were coded a three had evidence of students selecting the proper procedure and being able to arrive at the correct solution even though minor mistakes were made throughout the process. These minor mistakes are often important conceptually but do not impact the procedure of the problem when reaching the solution, as evidenced by a student's work in the example mentioned previously.

Of the 329 writing samples collected for the study, 9.7% of the samples were coded a two for procedural understanding. In written responses that were coded a two, students selected the appropriate procedure but made mistakes during the process causing an incorrect solution. There was an obvious difference when comparing student responses that were coded a three and student responses that were coded a two. While students with written responses that were coded a three skipped steps and made minor mistakes but still reached the correct solution, students with written responses that were coded a two made errors by just not understanding the procedure rather than simply skipping steps, as can be seen with the previously discussed example involving repair costs. She consistently made mistakes in her execution when trying to find total costs for different plans using the statistical concept of expected value.

Of the 329 writing samples collected for the study, 4.3% of the samples were coded a one for procedural understanding. There were two possibilities for student work that was coded a one. Students knew what procedure to use and could not begin the process, or they did not know the appropriate procedure. Written responses collected for this study that were coded a one for procedural understanding consisted of wrong approached being selected and mistakes when trying to carry out the wrong approach, which can be seen in a student's written response involving a one-proportion z-test. It was difficult to determine when she went from one step to another. She also incorrectly determined values and then misused those values. Written responses that were coded a one had nothing in common with written responses that were coded a two.

Sub-question #4

How can one describe the mathematical content in students' writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score. For Writing Prompt #2, see Appendix B, students were asked to analyze scores on the Graduate Record Examinations for a psychology department. Students are then asked to find the minimum score a student would need in order to score in the top 10% of those taking the test for this particular psychology department?

Using a rubric designed by Pugalee (2005), a student scored a four for the problem-solving process of mathematical content if the mathematics were accurate, all mathematical concepts and ideas were accurately identified, and mathematical terms were used appropriately. A student scored a four for problem-solving process of mathematical content as she solved this problem. She identified the formula for a z-score, $z = \frac{\bar{x} - \mu}{\sigma}$. She used the correct numbers in the correct places and concluded the correct solution. She wrote:

> The formula is $z = \frac{\bar{x} - \mu}{\sigma}$. $\mu$ and sigma are given, so I can plug in: $1.282 = \frac{\bar{x} - 544}{103}$. This answer is 676.046, so that is the minimum score in oreder to be in the top 10%...To be in the top 10% of those taking the GRE, a student must score at least 676.046.

Her mathematics was accurate. She substituted the given data in the correct places, enabling her to reach the appropriate solution.

For Writing Prompt #12, see Appendix L, students were given information regarding a shirt retailer. They were asked to use probability answer various questions. Using a rubric designed by Pugalee (2005), a student scored a three for the problem-solving process of mathematical content if the mathematics were accurate, mathematical concepts and ideas were accurately identified, and mathematical terms were used appropriately, but there were minor errors. A student scored a three for problem-solving process of mathematical content as she solved this problem.

The question asked students to find the proportion of customers who will be unable to find shirts in their sizes from this specific shirt retailer. After drawing the correct pictures to solve the problem, She wrote:

> $P(X < 14 \, and \, X \geq 18) \ldots z = \frac{\bar{x} - \mu}{\sigma} = \frac{14 - 15.7}{.7} = -2.43$.
> $z = \frac{\bar{x} - \mu}{\sigma} = \frac{18 - 15.7}{.7} = 3.29$.
> $P(X < 14) = .008 \, and \, P(X \geq 18) = .005$.
> .008+.005=.013.

She identified the correct mathematical concepts that were needed to solve this problem. Her steps as she calculated the solution was flawless, except for one minor error. She identified the $P(X \geq 18) = .005$, when it should have been $P(X \geq 18) = .0005$. This was likely a careless error. I circled this on Liz's paper without an explanation. When she received her paper back the next day, she brought it to me and said that she calculated correctly on the calculator, but incorrectly transferred the wrong solution. She said, "I missed a 0." She was frustrated, but it was clear to me that she understood the mathematical content of the problem.

For Writing Prompt #3, see Appendix C, students investigated male and female long jumpers. Students were asked to ascertain which jumper was more impressive, within their respective groups. Using a rubric designed by Pugalee (2005), a student scored a two for the problem-solving process of mathematical content if the mathematics contained minor errors, mathematical concepts and ideas were identified but with minor errors, and there were notable errors in the use of mathematical terms.

A student scored a two for problem-solving process of mathematical content as he solved this problem. After drawing the correct illustrations for the problem, he wrote:

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{275 - 263}{14} = .8571$$

$z = .8571$

85.71 percentile

$$z = \frac{\bar{x} - \mu}{\sigma} = \frac{207 - 201.2}{7.7} = .7532$$

$z = .7532$

75.32 percentile

Joey did better than Carla within each of their groups. He placed in about the 85$^{th}$ percentile (which means he did better than 85% of all other state college men long jumpers). While Carla only placed in the 75$^{th}$ percentile.

This student drew the correct conclusion that Joey had the more impressive jump within their respective groups. His reasoning when drawing that conclusion was flawed. He correctly calculated the z-scores for each jumper. However, there was a notable error when using the term "percentile." He could have used z-scores to explain his answer without even needing to use percentiles. He chose to use percentiles in his justification of his answer, which is acceptable had he done so correctly. At this point, it was clear that he believed that it was acceptable to use z-scores and percentiles synonymously. While students can use z-scores to obtain appropriate percentiles, the two concepts are related, but not the same.

For Writing Prompt #10, see Appendix J, students were introduced to a game involving two fair dice. These dice were numbered differently on their faces than the standard one through six on normal dice. There are two players with each one rolling a different die. The player with the higher number wins. Students are asked a couple of questions regarding probability, expected value, and the fairness of the game. Using a rubric designed by Pugalee (2005), a student scored a one for the problem-solving process of mathematical content if the mathematics was mostly inaccurate, mathematical concepts and ideas are identified with several errors, and mathematical terms are used inappropriately.

A student scored a one for problem-solving process of mathematical content as she solved this problem. The first part of the writing prompt asked students which die should be chosen to win the game. She wrote:

> I would select Die B because either way you role Die B, player 2 can only win if they roll a 9. You have a chance of winning either way with Die B. You must roll a 9 with Die A to win the game.

A correct way to have approached this problem included drawing a probability distribution table and calculating the expected values for each die. This process would have led her to the appropriate conclusion that Die A would have been better. She failed to mention any statistical concepts, except the term "chance."

A second part of this prompt supposed that the player using Die A would receive 45 tokens each time he won. The question then asked students to ascertain the number of tokens that the player using Die B would need to win each time to make this game fair. After drawing tree diagrams, that were incorrect, to try to find probabilities of winning for the player using each die, she wrote:

> Die A = WLLL = 25% chance of winning.
> Die B = WWLW = 75% chance of winning.
> .25(45) = 11.25
> .75(x) = 11.25
> x = 15
> 15 chips
> I found the probability of winning using Die A and Die B, and then I found the "break even" point for Die B.

The mathematical content used in her solution contained many errors. A correct solution included finding all of the possible outcomes of which die wins to help determine the probability of winning with each die. After the probabilities were found, students could use probability distribution tables and the concept of expected value to help determine the correct solution of 36 tokens. It is clear that Parker found 25% using division; she divided 1 by 4 for the one win out of four total games. While this is incorrect, it is unclear why she chose a total of four games. The total number of games was not stated in the problem because it had no relevance in solving the problem. When she found that .25(45) = 11.25, she then set .75(x) equal to 11.25. To complete this problem correctly, the expected values for each die need to equal. While she never notated or mentioned any concept regarding expected values, it appears that she was trying to obtain it because she knew that they were supposed to equal to make the game fair. The mathematics that she used contained many errors. Thus, she received a score of one for the problem-solving process of mathematical content.

Using a rubric designed by Pugalee (2005), a student scored a zero for the problem-solving process of mathematical content if the student showed no answer, or the mathematics has no relationship to the task. No one that participated in the study scored a zero for the problem-solving process of mathematical content.

Conclusion for Sub-question #4

Of the 329 writing samples collected for the study, 76.9% of the samples were coded a four for mathematical content. Students whose written responses were coded a four made sure to use mathematical terms correctly and properly identify mathematical concepts involved in problems. The mathematics involved when solving problems was accurate. This is evidenced by the student's response that included no flaws when determining what score would place students in the top 10% for GRE.

Of the 329 writing samples collected for the study, 11.2% of the samples were coded a three for mathematical content. Student responses that were coded a three or four were similar with one difference, the minor errors. Students whose written responses were coded a three were able to correctly identify mathematical terms and concepts but made minor mistakes when applying them, which can be seen in how the student in the previous example that was coded a three knew which method to use and how to calculate the probability correctly. However, she made a careless error when transferring an answer from her calculator to paper. The mathematics provided by the students when solving problems was accurate.

Of the 329 writing samples collected for the study, 8.2% of the samples were coded a two for mathematical content. Students whose written responses were coded a two made minor errors when identifying the mathematical concepts and using mathematical terms. Also, the mathematics when solving problems contained minor errors, as a student did when using z-score values and percentiles synonymously. His response demonstrated that he had a partial understanding of z-scores and percentiles found using z-scores, just not a complete understanding.

Of the 329 writing samples collected for the study, 3.6% of the samples were coded a one for mathematical content. Students whose written responses were coded a one involved numerous mistakes when identifying mathematical concepts and using mathematical terms. The mathematics when solving problems was mostly inaccurate. Student responses with this code were different from responses that were coded a two in that there was little to no evidence of understanding what mathematical concepts to apply, which is evidenced by the student's response when faced with a problem involving probability distribution tables and expected value. She failed to mention either statistical concept in her attempt to complete the problem.

Sub-question #5

How can one describe the mathematical reasoning in students' writings as they solve problems in a statistics course?

To answer this question, we will first look at examples of student work for each score. For Writing Prompt #1, see Appendix A, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, *A* or *B*, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable.

Using a rubric designed by Pugalee (2005), a student scored a four for the problem-solving process of mathematical reasoning if the student completely and accurately provided justification for major steps or processes, and defended the reasonableness of the answer with supporting reasons. A student scored a four for problem-solving process of mathematical reasoning as she solved this problem. After making a probability distribution table, She wrote:

> E(X) = 0(.5) + 1(.25) + 2(.15) + 3(.1)
> .85 ← expected # of repairs per year
> (3*.85)*$200 = $510 in three years + $10,500
> Machine A: $11,800 in 3 years
> Machine B: $11,010 in 3 years
> I began by addressing machine A, and I found out that with a monthly repair cost of $50 for three years, that it would cost $1800. When added to the cost for the machine, it would costs $11,800. I found the expected amount of times the machine would need repairs in a year for 1 year, and then found how many in three years, and then costing $200 a repair. The repairs in three years on Machine B would be $510. When the repair cost is added to the cost of Machine B, it would costs $11,080. Therefore, we would choose Machine B.

She made no mistakes while solving this problem. It is obvious from her explanation that she understood the basis of this problem as she provided key justifications for each step. Notice how she stated that she found the "expected amount of times the machine would need repairs" in her explanation. This statement made it clear that she understood the statistical concept in the context of the situation. She leads the researcher through her journey as she solved the problem and why she took each particular step. She left no doubt to the researcher that her mathematical reasoning was sound.

For Writing Prompt #15, see Appendix O, students investigated footprint data that had been gathered by anthropologists. The sizes of footprints were studied to estimate the size of the people who dwelled in the caves. The anthropologists wanted to construct a 95% confidence interval of the mean foot length for the adults. Students were asked to identify the assumptions that were necessary to make this confidence interval appropriate. Using a rubric designed by Pugalee (2005), a student scored a three for the problem-solving process of mathematical reasoning if the student accurately provided justification for major steps or processes but lacked clarity or detail, and defended the reasonableness of the answer but had minor omissions or errors in describing the approach.

A student scored a three for problem-solving process of mathematical reasoning as he solved this problem. He wrote:

> We must assume that the data on foot print size follows a normal distribution, also the sample must be from the population of interest, and finally the data must be from an SRS. The box plot for the footprint size appears to be skewed to the right. This suggests that the data may not follow the normal distribution. Therefore, it would be better to hesitate when drawing conclusions to the data. The problem states that the sample is from all footprints in the cave. This proves that the sample is from the population of interest. So this condition is met. The problem says that the 20 footprints were randomly selected, however, it does not state whether or not it was an SRS or not. Therefore, it would be best to hesitate when drawing conclusions to this confidence interval.

He wrote the data must come from a simple random sample (SRS) of the population of interest. This was correct and enabled the researcher to see that he understood one of the big concepts of the problem. The second assumption is that the data must come from a population that follows a normal distribution. It is with this concept that his mathematical reasoning lacked clarity and contained an error. He indicated that the data must follow a normal distribution, when the population that the data come from should follow the normal distribution. While this seems to be a common mistake among students, it was easy to recognize his misunderstanding through writing.

A writing sample from a student for the previous prompt, Writing Prompt#15, will be used to discuss and analyze a score of two. Using a rubric designed by Pugalee (2005), a student scored a two for the problem-solving process of mathematical reasoning if the student provided justification for most of the steps or processes with no errors, defended the reasonableness of the answer, but may not have developed supporting reasons for the answer.

A student scored a two for problem-solving process of mathematical reasoning as he solved this problem. He wrote:

> The assumptions necessary in order for the confidence interval to be appropriate are the following:
>
> 1. The sample must be an SRS (Simple Random Sample) from the population of interest. This would be stated in the problem or strongly implied.
> 2. The data must be normal or have a sample size great enough to use the Central Limit Theorem. The theorem states that as the sample size gets bigger it begins to approach a normal distribution. Normality is found by either being stated in the problem or by testing data if it is not given.
>
> For this problem:
>
> 1. An SRS from the population of interest was shown in the problem. The population of interest being adult human footprints in the prehistoric cave dwellings.
> 2. Based upon the 5 number summary, the data appears to be skewed to the right. So, we will hesitate when drawing conclusions.

He did not completely develop supporting reasons for part of his solution. He indicated that the five-number summary enabled him to determine if the data was skewed to the right. He did not provide justification as to how he determined this fact. Did he use the five-number summary to create a box plot? Did he use the five-number summary and the other data given in the problem to recognize that the mean was larger than the median, which would have helped determine that the data was skewed to the right? These are questions that cannot be answered when looking at his writing. If he would have drawn a box plot, that would have better supported his conclusion of the data.

For Writing Prompt #10, see Appendix J, students were introduced to a game involving two fair dice. These dice were numbered differently on their faces than the standard one through six on normal dice. There are two players who each role a different die. The player with the higher number wins. Students are asked questions regarding probability, expected value, and the fairness of the game. Using a rubric designed by Pugalee (2005), a student scored a one for the problem-solving process of mathematical reasoning if the student provided some justification for steps or processes but the response contained numerous errors and limited or no supporting evidence defending reasonableness of answer.

A student scored a one for problem-solving process of mathematical reasoning as she solved this problem. She wrote:

Die A

| X | 9 | 0 |
|---|---|---|
| P(X) | 0.66 | 0.33 |

Die B

| X | 3 | 11 |
|---|---|---|
| P(X) | 0.66 | 0.33 |

> Possible Combos
> 9,3; 9,11; 0,3; 0,11
> Die A wins!
> There is a higher chance of winning with Die B than with Die A. So I would choose Die B. The chance of winning with Die A is .444 (roll a 9 when other rolls a 3). With Die B, there are 3 different possible combinations that would result in a win.

It was apparent that she truly did not understand this problem through her writing and probability distribution tables. She provided justification for her steps; yet, her justification for steps was incorrect and contained many errors. For instance, she claimed that the person using Die A would win. In her next sentence, she claimed that the person using Die B had a higher chance of winning. This is a contradiction that she did not realize as she completed the problem. The probability distribution tables that she created were correct, but she did not make the connection that the statistical concept of expected value should be used to complete that portion of the problem. Instead, she wrote possible outcomes if the game was to be played. This enabled her to correctly calculate the probability of the person using Die A of winning. While she suspected that the probability of the person using Die B of winning was higher because there were more winning combinations, she was unable to calculate the actual probability for comparison.

Using a rubric designed by Pugalee (2005), a student scored a zero for the problem-solving process of mathematical reasoning if the student did not attempt to provide any justification for steps or processes. No one that participated in the study scored a zero for the problem-solving process of mathematical reasoning.

Conclusion for Sub-question #5

Of the 329 writing samples collected for the study, 78.7% of the samples were coded a four for mathematical reasoning. Students whose written responses were coded a four were able to clearly justify major steps and accurately defend the rationality of answers when needed. Students typically were able to take their solutions and contextualize their results, thus providing clear evidence of mathematical reasoning. The student from a previous example provided a complete response with no errors and justification for her reasoning that enabled the reader to know that she completely understood the problem.

Of the 329 writing samples collected for the study, 7.3% of the samples were coded a three for mathematical reasoning. Students whose written responses were coded a three provided evidence that students grasped both the major steps and minor steps of problems with minor mistakes. The student who wrote the response that was coded a three made a minor mistake with his reasoning when discussing a condition for a significance test regarding normal distributions. It is a common mistake that students make in class and on the Advanced Placement examination for statistics. Student responses that did not make the minor mistakes were coded a four.

Of the 329 writing samples collected for the study, 10.3% of the samples were coded a two for mathematical reasoning. Written responses that were coded a two were composed of solutions that had been justified without the development of supporting reasons. The student who wrote the response that was coded a two demonstrated this lack of justification by not clearly explaining his reasoning and how he used a five-number summary to draw conclusions. The primary difference for these responses compared to responses that were coded a three was that students provided some evidence of mathematical reasoning for the big picture of the problem with little evidence provided regarding the details.

Of the 329 writing samples collected for the study, 3.6% of the samples were coded a one for mathematical reasoning. Students whose written responses were coded a one were able to justify some steps but contained many errors. Students were not able to clearly demonstrate the majority of steps in a problem, and their written responses also did not show any defense of solutions. This can be seen when the student who wrote the response that was coded a two tried to determine if a person using Die A has a better chance of winning than a person using Die B. She claimed one person had a better chance using a Die A had a better chance, but in her next sentence concluded that the person using Die B had a better chance of winning. It was these contradictions combined with not knowing how to solve the problem and lack of reasoning with her approach that earned her a score of one.

Sub-question #6

How do the students' writings improve over time?

Low-scoring responses from students lacked a complete and detailed description as they solved problems, and students receiving low scores also failed to communicate through writing their understanding of the statistical concepts. Student work of such quality usually received scores of one or two using the rubric. Over time, as the study progressed, the writing of one student improved and became more descriptive. For this sub-question, an analysis of Phoebe's work at the beginning of the study, the middle of the study, and the end of the study will occur. At the beginning of the study, Phoebe's writing was not descriptive. As the study progressed, Phoebe's writing provided enriching responses as she solved problems. Phoebe is a student who loved English class in high school because she loved to read and write. She struggled to write in a statistics course at the beginning of the study. In time, she came to enjoy writing in a statistics course.

For Writing Prompt #2, see Appendix B, students were asked to analyze scores on the Graduate Record Examinations for a psychology department. Students are then asked to find the minimum score a student would need in order to score in the top 10% of those taking the test for this particular psychology department? To solve this problem, Phoebe started by drawing a picture of a normal distribution curve. While drawing pictures can be an important tool when solving problems, Phoebe unfortunately drew an incorrect picture for this problem. Next, she identified and wrote the formula for a standardized z-score. Phoebe substituted the correct values for the mean and standard deviation, but she substituted an incorrect value for the z-score. Phoebe wrote:

$$z = \frac{\bar{x} - \mu}{\sigma} \qquad .1 = \frac{\bar{x} - 544}{103}$$

10.3 = x – 544

554.3 = x

A student would need at least a 554.3, so we would say a minimum score of 555.

Phoebe should have substituted a value of 1.28 for the z-score. If she had drawn a correct picture of a normal distribution curve including a mark with the area to the left of the mark identified as .90 and the area to the right of the mark identified as .10, she likely would have found the appropriate value of 1.28 for the standardized z-score. At this point, it is clear that Phoebe scored a two for conceptual understanding as she displayed minor errors in logic and understanding when she displayed an incorrect picture for this problem. Phoebe scored a two for procedural understanding because she selected an appropriate approach by using the formula for a z-score but had a flawed execution by not correctly identifying the correct z-score of 1.28. Phoebe scored a two for problem-solving ability since she correctly identified the goal of this problem but misinterpreted a primary component of this problem.

Phoebe began to display a deeper knowledge of statistical concepts towards the middle of the study. For evidence of this claim, Phoebe's work for Writing Prompt #9, see Appendix I, will be examined. The problem gave students a salary schedule for teachers in a Midwest school district. They were asked to label a probability distribution table, find the mean salary of teachers in this district, find the mean teacher's contribution to their retirement plan, and write an article involving the information from the problem.

Phoebe correctly produced a probability distribution table for the teachers in this Midwestern district. She also found the correct mean salary of $36,235 and the correct mean contribution of $2311.75 that a teacher makes for retirement. Phoebe wrote:

| X ($) | 25,000 | 28,000 | 35,000 | 45,000 | 55,000 |
|---|---|---|---|---|---|
| P(X) | 0.105 | 0.245 | 0.345 | 0.21 | 0.095 |

(25,000)(.105) + (28,000)(.245) + (35,000)(.345) + (45,000)(.21) + (55,000)(.095)
2625 6860 12075 9450 5225
$\mu_x = \$36{,}235$
$(36,235)(.05) + $500 = $2311.75

Phoebe demonstrated a score of four for procedural understanding selecting and executing the appropriate strategy for each part of the problem. She also demonstrated a score of four for conceptual understanding when she identified that the primary statistical concept of the problem was expected value when she used the correct notation for expected value, $\mu_x$. Phoebe scored a four for mathematical content since she showed the proper steps correctly as she solved the different parts of this problem.

Phoebe did not completely display her statistical knowledge when she completed the part of the problem that had her write an article for the local newspaper. Phoebe wrote:

> Are School Teachers Underpaid?
> According to recent statistical analysis, the average pay of teachers per year is a mere $36,235.
> Why is it that teachers, who perform some of the most jobs in America, make an average yearly pay that is barely enough to live on?
> In fact, the probability that a teacher earns only $25,000 a year is .105 and only $28,000 a year is .245.
> This is horrendous. The maximum salary attainable is $55,000 and the probability that a teacher earns this is only .095. Although it is true that a teacher

> must have more experience to earn the $55,000 a year, it is horrible that the maximum salary is $55,000.
> Shows our priorities, doesn't it, America?

While Phoebe wrote the article with correct facts, she did not completely explain the probabilities. She also did not even mention the retirement plan and how it plays a role for teachers. She lacked a complete analysis of the problem when writing the article for the newspaper.

Phoebe showed more enriching responses as she completed the writing prompts near the end of the study. To see how Phoebe improved, her work for Writing Prompt #22, see Appendix V, will be examined. The problem gave students information regarding a football quarterback. Students were then asked to discuss and perform a simulation of twenty passes involving the quarterback. Students were also asked to find how many passes the quarterback is likely to throw before he completes a pass. Afterwards, each student was to write the football coach of the team that this quarterback plays and inform him how he can use this statistical information in a game.

The quarterback completes 44% of his passes. Phoebe successfully described and carried out a simulation of twenty passes. She also correctly identified the number of passes the quarterback would throw before a completion. Phoebe wrote:

> I would assign digits 00-43 as "making the pass" and 44-99 as "failing to pass".
> Then I would pick a row, say line 114, and select the first 20 numbers.
> Beginning on line 114:
> 71 54 60 52 <u>33</u> 53 94 66 87 <u>43</u> 72 46 <u>02</u> 76 <u>01</u> 45 <u>40</u> <u>38</u> 86 92
> 6/20 = .3
> 30 % successful
> Expected value = $\frac{1}{p} = \frac{1}{.44} = 2.27$

| X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| P(X) | 0.44 | 0.2464 | 0.138 | 0.077 | 0.043 | 0.024 |

Phoebe also demonstrated that she knew the primary statistical concept of the problem involved a geometric probability distribution when she chose the correct formula for expected value and created the geometric probability distribution table.

Phoebe further displayed her understanding of the problem in her letter to the coach. Phoebe wrote:

> Dear Coach,
> I have performed some statistical analysis on your quarterback's percentage of completed passes. He makes 44% of his passes. So, I created a geometric probability distribution. Using the formula for expected value of 1/p, I calculated that you can expect him to throw 2.27 passes before he completes one. So, you may want to wait until he is on his third pass to try anything vital to scoring.
> Hang in there Coach.
> - Phoebe

Phoebe basically told the coach that the quarterback is likely going to throw two incompletions before he makes his first completion. She not only demonstrated her knowledge of the statistical concept, but she also was able to correctly apply that to a specific "real-life" situation.

Conclusion for Sub-question #6

Phoebe's work was chosen to demonstrate how some students writing improved as the study progressed. She was a student who scored letter grades of B and C on her tests prior to the study. She was essentially an average to slightly above average student. It is clear that Phoebe's writing improved over the course of the study. This is evidenced by the comparison of her scores from the beginning of the study, the middle of the study, and the end of the study. The scores in Table 4.1 represent the scores that Phoebe's work was given for writing prompts two, nine, and 22.

Table 4.1: Phoebe's Scores for Written Prompts

| | Writing Prompt #2 | Writing Prompt #9 | Writing Prompt #22 |
|---|---|---|---|
| Problem-Solving Ability | 2 | 3 | 4 |
| Conceptual Understanding | 2 | 4 | 4 |
| Procedural Understanding | 2 | 4 | 4 |
| Mathematical Content | 2 | 4 | 4 |
| Mathematical Reasoning | 2 | 4 | 4 |

Over time, Phoebe began to better articulate through writing her understanding of statistical concepts. Notice that Phoebe showed growth in all areas of her problem-solving processes. Phoebe believed her growth in understanding statistical concepts was due to writing about them. She states, "Writing about something that you do not know about can be difficult. I like to write, and I like to write well. To do this, I needed to better understand what I was writing about. As I wrote, it forced me to learn about the concepts more. I found myself thinking about what I knew and did not know." This statement and reasons for Phoebe's growth will be further discussed in Chapter 6.

Sub-question #7

How can one describe problem posing by students as they solve problems in a statistics course?

English (1997) conducted a study involving problem posing, which she defined as giving students a numerical answer (p. 191) or a verbal statement (p. 192) and having students pose a problem after being given one of those options. Nine of the 24 writing prompts for this study had students solve a problem using statistical concepts. Afterwards, students were asked to pose a problem of a different context using the same statistical concepts. Unlike English, they were not given a numerical solution or a verbal statement as a starting place.

For one example of a student problem posing, a written sample from Writing Prompt #1, see Appendix A, will be used. Students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, *A* or *B*, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable.

Students were asked to pose a problem of a different context using the same statistical concept of the original written prompt. A student wrote:

> A pet lover is considering taking his dog to two different veterinarian offices in town. Vet office A charges a one-year membership fee of $250, while office B charges a membership fee of $300. The pet lover plans to sell his dog in two years when he goes off to college. A vaccination contract costs $10 per month at office A and $25 per month at office B. The distribution of the number of shots per year is as follows:

| # of shot | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| probability | 0.05 | 0.1 | 0.15 | 0.25 | 0.45 |

> The pet lover asks you to suggest which of the two he should go with. Which would you choose and why?

After reading this student's problem involving the statistical concept of expected value, I wondered why he had chosen a problem involving pets. When the response was returned, I asked him and he stated, "I love animals, and I spend a great deal of time reading about dogs." He took a statistical concept and posed a problem in a context of which he was familiar. It is clear that he understood the statistical concept of expected value when reading the problem that he posed. The process of problem posing assisted this student by helping cultivate his mathematical thinking and further develop his understanding, which is supported by Cifarelli and Sheets (2009).

For another example of a student problem posing, a written sample from Writing Prompt #3, see Appendix C, will be used. Students were asked to decide which long jumper, a male or female, had the better jump within their own groups. The primary statistical concept involved was finding z-scores and seeing which jumper was farther above the mean within their own group. Students were asked to pose a problem of a different context using the same statistical concept of the original written prompt. A student wrote:

> Scores for an IQ test are normally distributed for age groups. For the 15-30 years age group, the mean score is 115 with a standard deviation of 30. In the 50-70 years age group, the mean is 80 with a standard deviation of 20. Jessica is 18 years old and Jeanie is 62 years old. Jessica scored a 121 and Jeanie scored a 96. Who has a better score relative to their age group? Use statistical justification.

She included all of the pertinent information that a problem solver would need to solve this problem. When reading her written response, it was obvious that she knew the primary statistical concept of the original problem and was able to use the concept to create a problem of a different context. This was evidence that problem posing enabled this student to reflect on the statistical concept. Cifarelli and Sheets (2009) argue that problem posing involves reflection "that may stimulate the students' overall abilities to mathematize and develop understanding within new situations" (p. 245).

Not all of the student responses produced were questions of a different context. For instance, a student intended to write a question of a different context but it read more like a discussion of a different context. This student wrote:

> The best example of another way to apply these concepts of standardizing values is to take a mom's SAT score from the 1970s and her daughter's SAT score from

> the current year. The mom had an 1880, and the daughter had an 1830. Who scored better? Since these scores are from different eras, we need to standardize them to compare them. Before we say who scored better in their group, we have to find the z-scores. Then we can compare who had the higher score.

Technically, there is a question posed within her response. However, when this response is read as a whole, it appears to be a discussion. When trying to pose a problem, she demonstrated that she knew the main statistical concept involved in the original problem. An issue with her response would be that she did not include all of the pertinent information needed to solve her question. Her response was missing the means and standard deviations for both groups. Even though she excluded those two pieces of information, it appeared that the reflection process often involved in the process of problem posing still took place. She seemed to have a general sense of the main statistical concept as she contemplated and then wrote her response. This experience served to help build this student’s conceptual understanding of standardized values.

Having students problem pose about statistical concepts used in a problem and then create another problem of a different context involved a reflection process that helped deepen conceptual understanding for the student. More discussion involving problem posing will occur in Chapter 6.

# CHAPTER 5: QUANTITATIVE DATA ANALYSIS

## Research Question

Are there any differences between the problem-solving processes of problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning when analyzing written samples of students using ratings from a rubric?

## Delimitations and Limitations

Since a convenience sample was used, delimitations were present when trying to generalize the results. Gall, Gall, and Borg (2005) suggest that generalizations of results of a study are complicated when the participants are not randomly assigned. Results from this study can only be generalized to student populations similar to the population of the study.

With the exception of one possibility, limitations did not influence this study. Students' writing abilities were similar and did not range widely. This was determined prior to the beginning of the study using student grades in English and their score on the writing exam administered by the state of North Carolina.

When studying cognitive processes, it is a difficult task to try and measure as there is likely some overlap between the five problem-solving processes. When using HLM procedures, there is an expectation that there is little to no overlap of the processes being analyzed. This could be a possible limitation to this study.

HLM Results

The 24 questions for each of the five problem-solving facets that each student solved were nested within the 14 students. Therefore, the questions were at Level 1 since the differences of the five problem-solving facets were of interest to this study. The students were at Level 2 since that is how the problem-solving facets were arranged. Estimates of intra-class correlation (ICC), a ratio of the univariate between-student variance over the total variance, were calculated using the following formula (Stapleton, 2006), $ICC = \dfrac{MS_B - MS_W}{MS_B + (n.-1)MS_W}$. The estimates are presented in Table 5.1 for each construct with its mean and SD. Treating student as the grouping variable (n =14), multivariate analysis of variance (MANOVA) was used to obtain values for Mean Square Within ($MS_W$) and Mean Square Between ($MS_B$). Since there were 24 questions for each student, the first-level sample size was equal across students (n =24).

Table 5.1: Means, Standard Deviations, and Intra-class Correlation for Each Construct

| | Mean | SD | ICC |
|---|---|---|---|
| Problem-Solving Ability | 3.63 | 0.75 | 0.035 |
| Conceptual Understanding | 3.34 | 0.89 | 0.112 |
| Procedural Understanding | 3.61 | 0.83 | 0.101 |
| Mathematical Reasoning | 3.61 | 0.79 | 0.083 |
| Mathematical Content | 3.61 | 0.82 | 0.077 |

ICC values in Table 5.1 indicate that there was approximately 3.5% to 11.2% of variance at the student-level, justifying the use of hierarchical linear modeling (HLM), a multi-level analysis of data method by comparing mean differences at Level 1 data while considering that the Level 1 data are nested within Level 2 data (Raudenbush & Bryk, 2002). In doing HLM analyses, an unconditional model, when no predictors were added to the model, was run first to see if there was any variance to be explained for the outcome variable, which is each specific problem-solving process. A conditional model was run after examination of the estimation of the variance components in the unconditional model. The unconditional and conditional models are shown as follows:

Level 1:

$$Y_i = \pi_{0i} + \pi_{1i}X_{1i} + \pi_{2i}X_{2i} + \pi_{3i}X_{3i} + \pi_{4i}X_{4i} + e_i$$

where

$Y_i$ is the outcome variable (problem solving process) using the rubric in each of the five areas (e.g., rubrics in problem-solving ability) for student *I*;

$\pi_{0i}$ is the mean score on the outcome variable of student *I* when using the rubric of problem-solving ability;

$X_{1i}$ takes on a value of 1 when using the rubric of conceptual understanding;

$X_{2i}$ takes on a value of 1 when using the rubric of procedural understanding;

$X_{3i}$ takes on a value of 1 when using the rubric of mathematical reasoning;

$X_{4i}$ takes on a value of 1 when using the rubric of mathematical content;

$X_{1i}$, $X_{2i}$, $X_{3i}$, $X_{4i}$ takes on a value of 0 when using the rubric of problem-solving ability, respectively;

$\pi_{1i}$ is the difference on the outcome variable between using the rubric of conceptual understanding and problem-solving ability;

$\pi_{2i}$ is the difference on the outcome variable between using the rubric of procedural understanding and problem-solving ability;

$\pi_{3i}$ is the difference on the outcome variable between using the rubric of mathematical reasoning and problem-solving ability;

$\pi_{4i}$ is the difference on the outcome variable between using the rubric of mathematical content and problem-solving ability; and

$e_i$ is the residual of the model in Level 1.

Level 2: Each of the independent variables is used to predict the coefficients in the Level 1 Model. The following is the unconditional model without a predictor.

$$\begin{aligned} \pi_{0i} &= \beta_{00} + r_{0i} \\ \pi_{1i} &= \beta_{10} \\ \pi_{2i} &= \beta_{20} \\ \pi_{3i} &= \beta_{30} \\ \pi_{4i} &= \beta_{40} \end{aligned}$$

No predictors in Level 2 were used because none of the student characteristics was of interest to this study. This means that for the quantitative analysis, HLM focused only on possible differences between each problem-solving process. The coefficients for the differences between rubrics of each problem-solving process were fixed because the statements (hypotheses) were specific to the differences of the five areas of the rubric. The HLM analyses revealed statistically significant differences between conceptual understanding and that of problem-solving ability, $t$ (1643) = -9.231, $p < .001$, while all other estimated coefficients ( $X_{2i}$, $X_{3i}$, $X_{4i}$ ) were not statistically significantly different from zero. The magnitude of effect of the model, which was calculated by 1 minus the ratio between the standard error of the conditional model and that of the unconditional model, was 2%. This small percentage suggests that the model is reliable, and that the conclusions are legitimate.

## Conclusion

The only significant difference of the problem-solving processes found was between the conceptual understanding and problem-solving ability. These results suggest that when the researcher used the rubric to analyze student understanding that students received a significantly lower score for conceptual understanding compared to problem-solving ability. This result makes teachers aware of the fact that students need to further develop their conceptual understanding of statistical concepts as they solve problems.

# CHAPTER 6: DISCUSSION AND CONCLUSIONS

This chapter includes discussions on the major findings of the study. After restating the two primary questions, the findings from the analysis of data will be further discussed. This discussion will include how the methodology employed for this study provided a picture into student understanding that teachers do not otherwise have access. One major implication is how this picture enabled the researcher to better analyze student understanding and provide more relevant feedback to the students. Also, other teacher implications will be discussed and include diagrammatic literacy, a topic that was not originally sought by the researcher but appeared when students wrote in statistics. The chapter concludes with final recommendations by the researcher. As a reminder, the researcher was also the teacher for the students that participated in the study.

## Research Questions

Qualitative Question:

How do students' problem-solving processes through writing provide a rich description as they solve problems in a high school Advanced Placement Statistics course?

Quantitative Question:

Are there any differences between the problem-solving processes of problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning when analyzing written samples of students using ratings from a rubric?

## Discussion and Implications

The problem-solving processes included problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning. Chapter Four provided findings of each score for each problem-solving process. These were excerpts from written responses of students as they solved problems. By looking at these excerpts of each problem-solving process separately, it provided a picture of student understanding in individual parts. When combining these individual parts to view the whole picture, student understanding of statistical concepts became evident through writing.

The whole picture of student understanding of statistical concepts is provided by analyzing a written response specifically looking at five areas of problem solving, which include problem-solving ability, conceptual understanding, procedural understanding, mathematical content, and mathematical reasoning. The writing samples of students who tended to do well when providing their solutions to problems consistently scored threes and fours using the rubric. These students were successfully able to demonstrate the five problem-solving processes as they solved problems over the entire eight-week study. However, not all students were successful in this manner.

Utilizing a method involving writing provided a complete picture of student understanding when viewing the different problem-solving processes and enabled the researcher to recognize that one student named Phoebe showed tremendous growth in her conceptual understanding as the study progressed. Requiring Phoebe to write as she solved problems provided Phoebe and the researcher a complete picture of her understanding. Phoebe’s conceptual understanding of statistical ideas was the problem-solving process that became apparent and relevant. Thus, the researcher was aware of Phoebe's level of understanding and better assisted her with feedback that would help her be more successful. The feedback was written comments made on Phoebe's written responses to the problems that she solved. These comments led to discussions initiated by Phoebe to correct her misunderstandings.

By knowing exactly where she was making mistakes and then being able to correct the mistakes led to Phoebe enjoying the class more than she had been before the written prompts were utilized. She stated, "I actually enjoy writing. English is my favorite class because I get to write. I was unsure about writing in math since it is hard to write about something I do not know a lot about, but it ended up helping me become more successful." A major implication of this study is how the rich descriptions of student writing provide teachers with a complete representation of student understanding. This results in the teacher being better equipped to help students, like Phoebe, become more successful in statistics.

Phoebe was not the only student to benefit from writing in a statistics course. The conceptual understanding of statistical concepts for students is not always visible or easy for teachers to recognize when grading multiple-choice tests or short-answer tests primarily involving calculations. The use of writing as a communicative tool for students enabled the researcher to better see when students truly grasp statistical concepts. For example, based on observations as a teacher prior to this study, the researcher felt that another student had shown an adequate conceptual understanding of the statistical term called expected value. She was adept at this skill and had demonstrated on multiple-choice tests and short-answer tests that she could successfully find the expected value.

However, it became clear that she did not truly understand the concept of expected value when analyzing her work for Writing Prompt #1, see Appendix A. This supported one of the major findings of this study that there was a significant difference between conceptual understanding and problem-solving ability. Through discussions with educators, many of them believe that students have a difficult time solving problems in mathematics when conceptual understanding is limited. This study shows in statistics courses that students may have a limited understanding conceptually while still being able to solve a problem.

For this problem, students were asked to give a recommendation to a department based on overall cost as to which photocopy machine, *A* or *B*, along with its repair contract, should be purchased. This particular department replaces photocopy machines every three years. The primary statistical concept involved the process of calculating the expected value of a discrete random variable. A student wrote:

> I used the mean of a random variable. This showed me the mean of the variable as an average of the possible values of X, but with a change to consider that all outcomes don't need to be equal.
> $\mu_x = x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4$
> $= 0(.5) + 1(.25) + 2(.15) + 3(.1)$
> $= .85$, which is the probability you will need a repair

While the student identified the correct statistical concept for the problem, she was able to correctly calculate it, but she did not show that she completely knew what the concept meant and how to apply it to the problem. The value of .85 indicated the expected number of photocopy machine repairs for this problem. The value of .85 is not a probability that you will need a repair. The feedback to the students involved having her consider a different problem and calculate the expected value. Since the term probability was underlined in her original response and the new problem's expected value had a value of 1.75, she realized that the term expected value is not synonymous with probability, due to the fact that an event cannot have a probability greater than one.

Notice how her response was adequate regarding her procedural understanding. She cited a correct formula, substituted the appropriate values, and provided the correct numerical value of .85. This supports a previous statement that she could arrive at the correct solution and usually select the correct answer for a multiple-choice question that simply asked for the expected value. The student's response is representative of three other students' responses regarding conceptual understanding of the statistical concept of expected value.

The role of writing in a statistics classroom provided the researcher with a tool that enabled the researcher to identify conceptual understanding for students as they solved problems. It is the process of writing that required students to better understand the material that is being learned. For students to sufficiently articulate statistical concepts well in written form, they need to have a deeper understanding of those concepts. The role of writing helped students realize what they know and did not know. This supports another implication that teachers in statistics classrooms should implement some form of writing. The benefits of requiring students to write about their conceptual understanding regarding specific content are too significant to ignore.

Another finding using a HLM procedure revealed a statistically significant difference between using the rubric of conceptual understanding and that of problem-solving ability, $t$ (1643) = -9.231, $p < .001$, while all other estimated coefficients ( $X_{2i}$, $X_{3i}$, $X_{4i}$ ) were not statistically, significantly different from zero. These results suggest that students received a significantly lower score in mathematics problem solving for conceptual understanding compared to problem-solving ability. This finding emphasizes the importance of conceptual understanding of topics for students and suggests that the problem-solving ability of students while they solve problems in a statistics course will only increase as their conceptual understanding improves. How can teachers improve conceptual understanding for students in a statistics course? Problem posing is a method which can assist students in this endeavor and will be further discussed later in this chapter.

## Other Teacher Implications

While some teachers may be hesitant to step out of their comfort zone and employ a writing method that has students writing as they solve problems, this study demonstrated that it was beneficial for students as they learn statistical concepts. This study has also shown that requiring students to write in statistics class improved Advanced Placement Statistics exam scores. Writing in a statistics classroom is a tool for teachers that provides insight and helps make visible students' problem-solving processes through different facets, which has been discussed in Chapters Four and Five. There are three more teacher implications to be discussed in the following sections, which include the Advanced Placement Statistics exam, diagrammatic literacy, and problem posing.

### Advanced Placement Statistics Exam

The students who participated in this study took the Advanced Placement Statistics exam during May of 2010. A score of 3, 4, or 5 will earn students credit for the course at the college level. A score of 1 or 2 demonstrates that the student has not shown a proficient ability in statistics to earn college credit. The scores in Table 6.1 represent the scores of the students that participated in the study.

Table 6.1: Advanced Placement Exam Scores for Students in the Study

| Score |
|---|
| 5 |
| 5 |
| 4 |
| 4 |
| 4 |
| 4 |
| 4 |
| 4 |
| 4 |
| 4 |
| 3 |
| 3 |
| 3 |
| 3 |

No student scored below a 3 for the class. This is well above the expected values when comparing the students to the national averages. The information in Table 6.2 represents the students in this class compared to all of the students that took the exam nationally.

Table 6.2: Percentages of Student Scores for this Study and Nationally

| Score | Students in the Study | All Students Nationally |
|---|---|---|
| 5 | 14.3% | 12.8% |
| 4 | 57.1% | 22.4% |
| 3 | 28.6% | 23.5% |
| 2 | 0.0% | 18.2% |
| 1 | 0.0% | 23.1% |
| Proficient | 100.0% | 58.7% |
| Mean | 3.86 | 2.84 |

When looking at Table 6.2, it is clear that only 58.7% of students earned a proficient score nationally compared to 100% of the students that participated in the study involving the written method. Also, these students compared favorably with a mean score of 3.86

compared to 2.84 for all students. The concept of writing in statistics appears to have helped students understand the material better, which likely lead to better scores than would be expected when compared to the national data.

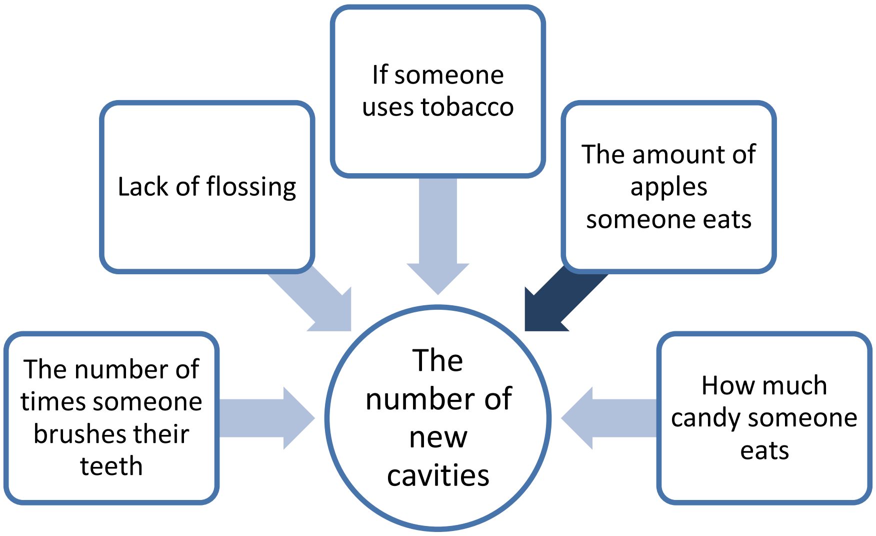
Within the same school, the scores showed improvement from previous years. There were 55.5% of students at the school that earned a proficient score in 2007, 92.4% in 2008, and 83.3% in 2009. Since 2008, writing has been used as a tool in learning, but it was not as intensive as the writing that took place during the study that resulted in 100% proficiency for 2010.

Diagrammatic Literacy

The writing also revealed that students utilized different forms of writing when solving problems. Diagrams were an important communicative tool for students as they solved problems. Students can use diagrammatical literacy as a way to communicate statistical concepts. It also provided the researcher with another technique when looking at the conceptual understanding of statistical concepts of students. For Writing Prompt #7, see Appendix G, students were given a problem involving dentists in a dental clinic who were studying a possible difference between the number of new cavities in people who eat an apple a day and in people who eat less than one apple a week. There were many parts to this question that students had to answer. One specific part of the question required students to explain a confounding variable in the context of the problem and then identify a possible confounding variable.

A student did not explain what a confounding variable was in the context of this study. She explained what a confounding variable was in a general sense. She wrote, “Two variables are confounded when their effects on a response variable cannot be distinguished from each other." She provided a chart giving possible confounding variables, see Figure 6.1. This enabled the researcher the opportunity to see that she understood the statistical concept.

Figure 6.1: A Student's Confounding Variables Chart




The dark arrow was drawn from the explanatory variable to the response variable. This indicated that the student knew that the dentists were claiming that the amount of apples someone eats might be responsible for the number of cavities that person has. The lighter arrows were drawn from possible confounding variables to the response variable. She indentified these possible confounding variables and wrote beneath her diagram:

> The amount of apples someone eats cannot specifically explain the number of new cavities they receive because there are other confounding variables, such as tobacco use or lack of flossing, that contribute to the number of new cavities that someone has. I do not believe that one could conclude that the lower number of new cavities can be attributed to eating an apple a day because there are so many other variables that could still affect this that you still cannot pin it to just from an apple a day.

This sample of student writing showed the student's conceptual understanding as she solved the problem. Perhaps, she used the diagram to help her formulate her thoughts as she solved the problem. She was then able to better elucidate her thoughts regarding the concepts of the problem. Students used diagrams to help solve problems and formulate thoughts regarding the statistical concepts. Developing a mathematical literacy in the classroom does not only involve students writing sentences. The use of diagrams and graphic organizers can also be critical for students in mathematics classrooms (Diezmann & English, 2001). This is another example of how requiring students to write in a statistics classroom provided insight into a facet of a student's problem-solving ability.

Problem Posing

Nine of the twenty-four problems involved a component that asked students to pose problems when given a specific statistical concept. For problem posing to successfully work, the idea is to help students use their own experiences to participate in mathematical activities (Silver, 1995). My study supports this belief in the statistics classroom. In Chapter Four, a written sample provided by a student was analyzed and discussed. He wrote about the statistical concept of expected value in a different context of the original problem. He successfully designed a problem about two veterinarians and office plans regarding pets. During a follow-up discussion with him, he discussed his love for dogs and that was why he chose that context. The student was able to use his own experience and deepen his understanding of a statistical concept as he posed problems. Students who are allowed and given the opportunity to use personal experiences and situations to pose problems will deepen their understanding of material in a statistics classroom.

There was only one issue that was noticed after analyzing the written responses several times after the study had been conducted was the nature of some of the problems posed by the students. When students were asked to pose problems involving a statistical concept, it was the last part of a written prompt that had them solve a problem using that same concept prior to posing their own problem. On different occasions, only three students wrote problems of a similar context to the original problem. They were unable to use a personal experience to assist in deepening an understanding of statistical material. This suggests that the students likely had a surface understanding of the statistical concept involved and felt more comfortable mimicking the original problem and only changing the numbers involved in the problem. Students have a hard time problem posing when they do not have any grasp of the statistical material, thus, causing students issues when going through the reflective process that is required when problem posing.

## Conclusion

Teachers are constantly trying to improve their classroom instruction to make class more meaningful for students and improve student understanding of concepts. This study has shown teachers of statistics the benefits of using writing as a tool to better understand the problem-solving abilities of students. While this study, consisting of an eight-week unit, had concluded before the Advanced Placement Statistics exams were administered, the students and the researcher discussed their feeling regarding the writing-intensive unit that they had just participated. All students agreed that they did not enjoy all of the writing involved with the unit. A student remarked, “I have to write enough in English class." Another student agreed, "In English class, I have to write. In history class, I have to write. I do not want to have to come to math class to write."

The students did appreciate how the writing prompts involved problems from statistical concepts that had been previously learned. A student stated, "I had forgotten some of those concepts. With the AP exam coming up and after having spent so much time writing about them, it felt like a good review and I understand them." When he said this during the discussion, twelve of the remaining thirteen students concurred.

Once students had taken the Advanced Placement Statistics exams, the researcher revisited the discussion of writing in a statistics. The students still agreed that they hated all of the "extra work" when completing the problems, but their tone as a class had changed. A student shared her viewpoint, "I felt so prepared for that exam. I truly believe that the writing prompts are what made the difference. I was talking to a friend at another school who said that she felt lost." The general consensus of the class, at this point, appreciated the purpose of writing in statistics, just did not appreciate doing the writing.

The other implications that arose from the findings of this study that have been previously mentioned in this chapter will be summarized. First, teachers should require students to write as they solve problems since the written responses will provide teachers with an enriching description of student understanding of statistical ideas. This enables teachers to better assist students with feedback that would help them be more successful. Through discussions with teachers who do not teach mathematics, they dislike how in mathematics that a solution is either right or wrong. The notion of having students write in a statistics course permits teachers to evaluate understanding on a higher level than just right or wrong. Second, the role of writing aids students to realize the depth of statistical material that they actually know. Third, requiring student to write in statistics leads to students understanding the material better, which leads to better scores on the Advanced Placement Statistics examination than would be expected when looking at the national data. The last implication involves problem posing and that this process requires students to reflect on a statistical concept, pull from personal experiences, and successfully write a problem of a different context that the original problem. This process requires students to clearly articulate their knowledge and leads to a deeper understanding.

It is my hope that mathematics educators that teach statistics will use my findings as evidence of more profound understanding of concepts and employ some form of writing in their classrooms. Developing a mathematical literacy should be a goal among all mathematics educators. While these results can only be generalized to similar populations, further research should be conducted to different populations, including populations that have participants that come from lower-income families. If similar results of future research regarding the suggested populations match my findings, it would lend credence and further support the role of writing in a statistics classroom.

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# APPENDIX A: WRITING PROMPT #1

A department supervisor is considering purchasing one of two comparable photocopy machines, *A* or *B*. Machine *A* costs \$10,000, and machine *B* costs \$10,500. This department replaces photocopy machines every three years. The repair contract for machine *A* costs \$50 per month and covers an unlimited number of repairs. The repair contract for machine *B* costs \$200 per repair. Based on past performance, the distribution of the number of repairs needed over any one-year period for machine *B* is shown below.

| Number of Repairs | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | 0.5 | 0.25 | 0.15 | 0.1 |

a) You are asked to give a recommendation based on overall cost as to which machine, *A* or *B*, along with its repair contract, should be purchased. What would your recommendation be? Give a statistical justification to support your recommendation.

b) Write a letter to a fictitious person that was absent from class today explaining the problem that you completed. Include the following components in your letter.

# APPENDIX B: WRITING PROMPT #2

The Graduate Record Examinations are widely used to help predict the performance of applicants to graduate schools. The range of possible scores on a GRE is 200 to 900. The psychology department finds that the scores of its applicants on the quantitative GRE are approximately normal with mean 544 and standard deviation 103.

a) What minimum score would a student need in order to score in the top 10% of those taking the test?

b) Write a letter to a fictitious person that was absent from class today explaining the problem that you completed. Include the following components in your letter.

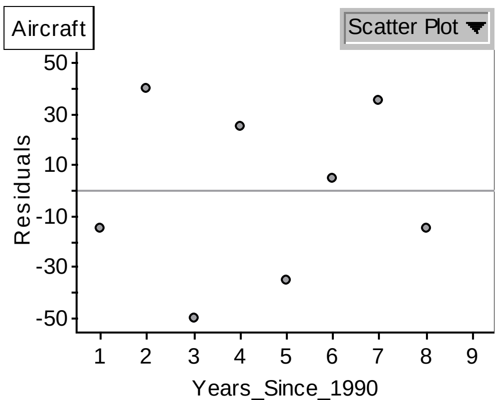
# APPENDIX C: WRITING PROMPT #3

The best male long jumpers for State College since 1973 have averaged a jump of 263.0 inches with a standard deviation of 14.0 inches. The best female long jumpers have averaged 201.2 inches with a standard deviation of 7.7 inches. This year Joey jumped 275 inches and his sister, Carla, jumped 207 inches. Both are State College students. Assume that male and female jumps are normally distributed.

a) Within their groups, which athlete had the more impressive performance? Explain using statistics to support your answer.
b) Identify the statistical concept(s) that you used to solve the problem and define each one.
c) Create a different problem using the same statistical concepts.

# APPENDIX D: WRITING PROMPT #4

Lydia and Bob were searching the Internet to find information on air travel in the United States. They found data on the number of commercial aircraft flying in the United States during the years 1990-1998. The dates were recorded as years since 1990. Thus, the year 1990 was recorded as year 0. They fit a least squares regression line to the data. The graph of the **residuals** and part of the computer output for their regression are given below.




$\hat{y} = 2939.93 + 233.517x$

r = 0.88

a) Is a line an appropriate model to use for these data? What information tells you this?
b) What is the value of the slope of the least squares regression line? Interpret the slope in the context of this situation.
c) What is the value of the intercept of the least squares regression line? Interpret the intercept in the context of this situation.
d) What is the predicted number of commercial aircraft flying in 1992?
e) What was the actual number of commercial aircraft flying in 1992?

f) Identify the statistical concept(s) that you used to solve the problem and define each one.
g) Create a different problem using the same statistical concepts.

# APPENDIX E: WRITING PROMPT #5

Animal-waste lagoons and spray fields near aquatic environments may significantly degrade water quality and endanger health. The National Atmospheric Deposition Program has monitored the atmospheric ammonia at swine farms since 1978. The data on the swine population size (in thousands) and atmospheric ammonia (in parts per million) for one decade are given below.

| Year | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
|---|---|---|---|---|---|---|---|---|---|---|
| Swine Population | 0.38 | 0.50 | 0.60 | 0.75 | 0.95 | 1.20 | 1.40 | 1.65 | 1.80 | 1.85 |
| Atmospheric Ammonia | 0.13 | 0.21 | 0.29 | 0.22 | 0.19 | 0.26 | 0.36 | 0.37 | 0.33 | 0.38 |

a) Construct a scatterplot for these data.
b) The value for the correlation coefficient for these data is 0.85. Interpret this value.
c) Based on the scatterplot in part (a) and the value of the correlation coefficient in part (b), does it appear that the amount of atmospheric ammonia is linearly related to the swine population size? Explain.
d) What percent of the variability in atmospheric ammonia can be explained by swine population size?

e) Describe how you solved this problem, step by step. How did you know to take each step. If you used the calculator for any part, make sure to explain why you used it for that step.

# APPENDIX F: WRITING PROMPT #6

Foresters are interested in predicting the amount of usable lumber they can harvest from various tree species. The following data have been collected on the diameter of Ponderosa pine trees, measured at chest height, and the yield in board feet. Note that a board foot is defined as a piece of lumber 12 inches by 12 inches by 1 inch.

| Diameter | Bd Feet |
|---|---|
| 36 | 192 |
| 28 | 113 |
| 28 | 88 |
| 41 | 294 |
| 19 | 28 |
| 32 | 123 |
| 22 | 51 |
| 38 | 252 |
| 25 | 56 |
| 17 | 16 |
| 31 | 141 |
| 20 | 32 |
| 25 | 86 |
| 19 | 21 |
| 39 | 231 |
| 33 | 187 |
| 17 | 22 |
| 37 | 205 |
| 23 | 57 |
| 39 | 265 |

a) Construct an appropriate model for these data. Then comment on the quality of your model.

c) Write a letter to a fictitious person that was absent from class today explaining, in depth, the problem that you completed and the statistical concepts that were used.

# APPENDIX G: WRITING PROMPT #7

The dentists in a dental clinic would like to determine if there is a difference between the number of new cavities in people who eat an apple a day and in people who eat less than one apple a week. They are going to conduct a study with 50 people in each group. Fifty clinic patients who report that they routinely eat an apple a day and 50 clinic patients who report that they eat less than one apple a week will be identified. The dentists will examine the patients and their records to determine the number of new cavities the patients have had over the past two years. They will then compare the number of cavities in the two groups.

a) Why is this an observational study and not an experiment?
b) Explain the concept of confounding in the <u>context</u> of this study. Include an example of a possible confounding variable.
c) If the mean number of new cavities for those who ate an apple a day was statistically smaller than the mean number of cavities for those who ate less than one apple a week, could one conclude that the lower number of new cavities can be attributed to eating an apple a day? Explain.
d) Design a problem using similar statistical concepts. Include the complete solution to your problem. Discuss how your problem is similar and different to this problem.

# APPENDIX H: WRITING PROMPT #8

A blood disease is found in 2% of the persons in a certain population. A new blood test will correctly identify 96% of the persons with the disease and 94% of the persons without disease.

a) What is the probability that a person does not have the disease?
b) What is the probability that a person is correctly identified?
c) What is the probability that a person who is identified as having the disease actually has the disease?
d) Describe the rules of probability that you used, their importance, and their mathematical importance.

# APPENDIX I: WRITING PROMPT #9

The salary schedule for teachers in a Midwest school district has five steps. Here are the salaries and number of teachers for each step.

| Step | Salary | Teachers |
|---|---|---|
| 1 | $25,000 | 42 |
| 2 | $28,000 | 98 |
| 3 | $35,000 | 138 |
| 4 | $45,000 | 84 |
| 5 | $55,000 | 38 |

a) Let X be the salary. Give the probability distribution of X.
b) What is the probability that a randomly chosen teacher earns more than $40,000?
c) What is the mean salary $\mu_x$ in the school district?
d) A teacher's contribution to the school district's retirement plan is $500 plus 5% of his or her salary. What is the mean contribution?
e) Write an article for your local newspaper using the information from the problem and its solutions. Note: Steps involve years of experience.

# APPENDIX J: WRITING PROMPT #10

Die A has four 9’s and two 0’s on its faces. Die B has four 3’s and two 11’s on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. two players are going to play a game. The first player selects a die and rolls it. the second player rolls the remaining die. The winner is the player whose die has the higher number on top.

a) Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.
b) Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.

   * A fair game is one in which the player using die A and the player using die B both end up with the same amount of tokens.

c) Describe what made this problem easy or difficult. Make sure to explain completely.
d) Describe, specifically, where a student could go wrong (make a mistake) when completing this problem.

# APPENDIX K: WRITING PROMPT #11

According to government data, 20% of employed women have never been married.

a) If 10 employed women are selected at random, what is the probability that 2 or fewer have never been married?

b) What is the random variable X of interest here? Define X. Is X normal, binomial, or geometric?

c) What are the mean and standard deviation of X?

d) Find the probability that the number of employed women who have never been married is within 1 standard deviation of its mean.

e) Describe the four conditions that describe a binomial setting.

f) What chapter of the textbook helped you answer these questions? After you have identified the chapter, pick only two of the main concepts and describe them in detail.

# APPENDIX L: WRITING PROMPT #12

Men's shirt sizes are determined by their neck sizes. Suppose that men's neck sizes are approximately normally distributed with mean 15.7 inches and standard deviation 0.7 inch. A retailer sells men's shirts in sizes S, M, L, XL, where the shirt sizes are defined in the table below.

| Shirt Size | Neck Size |
|---|---|
| S | $14 \le$ neck size $< 15$ |
| M | $15 \le$ neck size $< 16$ |
| L | $16 \le$ neck size $< 17$ |
| XL | $17 \le$ neck size $< 18$ |

a) Because the retailer only stocks the sizes listed above, what proportion of customers will find that the retailer does not carry any shirts in their sizes? Show your work.
b) Using a sketch of a normal curve, illustrate the proportion of men whose shirt size is M. Calculate this proportion.
c) Of 12 randomly selected customers, what is the probability that exactly 4 will request size M? Show your work.
d) Describe how technology, if possible, can help you solve this problem.
e) Design a problem using similar statistical concepts. Include the complete solution to your problem.

# APPENDIX M: WRITING PROMPT #13

A survey asks a random sample of 1500 adults in Ohio if they support an increase in the state sales tax from 5% to 6%, with the additional revenue going to education. Let *p* denote the proportion in the sample that says they support the increase. Suppose that 40% of all adults in Ohio support the increase.

a) If $\hat{p}$ is the proportion of the sample who support the increase, what is the mean of $\hat{p}$?
b) What is the standard deviation of $\hat{p}$?
c) Explain why you can use the formula for the standard deviation of $\hat{p}$ in this setting.
d) Check that you can use the normal approximation for the distribution of $\hat{p}$.
e) Find the probability that $\hat{p}$ takes a value between 0.37 and 0.43.
f) Write a question, of a different context, using the same statistical principles. Give the answer to each part of your question.

# APPENDIX N: WRITING PROMPT #14

A friend who hears that you are taking a statistics course asks for help with a chemistry lab report. She has made four independent measurements of the specific gravity of a compound. The results are 3.82, 3.93, 3.67, and 3.98. The lab manual says that repeated measurements will vary according to a normal distribution with standard deviation $\sigma =$ 0.15. (This standard deviation shows how precise the measurement process is.) The mean $\mu$ of the distribution of measurements is the true specific gravity. The lab manual also asks whether the data show that the true specific gravity is less than 3.9. To assess this, test the hypotheses [ $H_0 : \mu = 3.9$] [ $H_a : \mu < 3.9$] .

a) First calculate the test statistic, then find the P-value.
b) What do you conclude?
c) Compose an email to the friend explaining, in layman's terms, if the data show that the true specific gravity is less than 3.9. In this email, make sure to also explain what might make your conclusions better if you were asked to do this analysis again. Limit your email to five sentences. They need to succinct and thorough.

APPENDIX O: WRITING PROMPT #15

Anthropologists have discovered a prehistoric cave dwelling that contains a large number of adult human footprints. To study the size of the adults who used the cave dwelling, they randomly selected 20 of the footprints from the population of all footprints in the cave and measured the length of those footprints. Some statistics resulting from this random sample are as follows:

| | | | |
|---|---|---|---|
| Sample size | 20 | Minimum | 15.2 cm |
| Mean | 24.8 cm | First quartile | 18.7 cm |
| Standard deviation | 7.5 cm | Median | 21.5 cm |
| | | Third quartile | 30.0 cm |
| | | Maximum | 37.0 cm |

The anthropologists would like to construct a 95 percent confidence interval for the mean foot length of the adults who used the cave dwelling.

a) What assumptions are necessary in order for this confidence interval to be appropriate?
b) Discuss, in detail, whether each of the assumptions listed in your response to (a) appears to be satisfied in this situation.

# APPENDIX P: WRITING PROMPT #16

Patients with chronic kidney failure may be treated by dialysis, using a machine that removes toxic wastes from the blood, a function normally performed by the kidneys. Kidney failure and dialysis can cause other changes, such as retention of phosphorous, that must be corrected by changes in diet. A study of the nutrition of dialysis patients measured the level of phosphorous in the blood of several patients on six occasions. Here are the data for one patient (milligrams of phosphorous per deciliter of blood):

| | | | | | |
|---|---|---|---|---|---|
| 5.6 | 5.3 | 4.6 | 4.8 | 5.7 | 6.4 |

The measurements are separated in time and can be considered an SRS of the patient's blood phosphorous level. Assume that this level varies normally with $\sigma = 0.9$ mg/dl. The normal range of phosphorous in the blood is considered to be 2.6 to 4.8 mg/dl.

a) Is there strong evidence that the patient has a mean phosphorous level that exceeds 4.8?

b) Describe a Type I error and a Type II error in this situation. Which is more serious?

c) Give two ways to increase the power of the test you performed in (a).

d) Write two or more paragraphs describing your steps and why you took those steps as you completed part (a).

# APPENDIX Q: WRITING PROMPT #17

The Colorado Rocky Mountain Rescue Service wishes to study the behavior of lost hikers. If more were known about the direction in which lost hikers tend to walk, then more effective search strategies could be devised. two hundred hikers selected at random from those applying for permits are asked whether they would head uphill, downhill, or remain in the same place is they became lost while hiking. Each hiker in the sample was also classified according to whether he or she was an experienced or novice hiker. The resulting data are summarized in the following table.

| | **Direction** | | |
|---|---|---|---|
| | Uphill | Downhill | Remain in Same Place |
| **Novice** | 20 | 50 | 50 |
| **Experienced** | 10 | 30 | 40 |

a) Do these data provide convincing evidence os an association between the level of hiking expertise and the direction the hiker would head if lost? Give appropriate statistical evidence to support your conclusion.

b) Write a letter to a fictitious person that was absent today. This needs to be an **in-depth** analysis.

# APPENDIX R: WRITING PROMPT #18

A random sample of 200 students was selected from a large college in the United States. Each selected student was asked to give his or her opinion about the following statement. "The most important quality of a person who aspires to be the President of the United States is a knowledge of foreign affairs."

Each response was recorded in one of five categories. The gender of each selected student was noted. The data are summarized in the table below.

| Response Category | | | | | |
|---|---|---|---|---|---|
| | Strongly Agree | Somewhat Agree | Neither Agree nor Disagree | Somewhat Agree | Strongly Agree |
| Male | 10 | 15 | 15 | 25 | 25 |
| Female | 20 | 25 | 25 | 25 | 15 |

a) Is there sufficient evidence to indicate that the response is dependent on gender? Provide statistical evidence to support your conclusion.
b) Write a newspaper column for USA Today describing the poll. Remember that newspapers are typically written on a sixth-grade level, and that the majority of your sentences must be in layman's terms.

# APPENDIX S: WRITING PROMPT #19

A large university provides enough housing for 10 percent of its graduate students to live on campus. The university's housing official thinks that the percent of graduate students looking for housing on campus may be more than 10 percent. The housing official decides to survey a random sample of graduate students, and 62 of the 481 respondents say they are looking for housing on campus.

a) On the basis of the survey data, would you recommend that the housing office consider increasing the amount of housing on campus available to graduate students? Give appropriate statistical evidence to support your recommendation.
b) In addition to the 481 graduate students who responded to the survey, there were 19 who did not respond. If these 19 had responded, is it possible that your recommendation would have changed? Explain.
c) Write a released statement on behalf of the university that will be sent to the incoming graduate students describing the survey and the results. This needs to be well-written and polished, considering that you are representing the university.

# APPENDIX T: WRITING PROMPT #20

In the drought year 1988, statements were made that over half of Indiana corn producers did not get back from their corn crop the money they put into seed, fertilizer, etc. To check this, a random sample of 800 farms is chosen and a brief audit is made on each of these farms. Of these farms, 405 did not recover their costs from their corn crops.

a.) Is this good evidence for the claim in the first sentence?

b.) Is there sufficient evidence at the 5% level to support the claim made in the first sentence? Give details regarding how you arrived at your conclusion.

c.) Give a 95% confidence interval for the proportion of producers who did not get back their money.

d.) Write a newspaper article for USA Today describing the state of farms in Indiana in 1988. Describe the "basic" statistics, remember it is a newspaper, and whether the farming industry in Indiana is worth the risk. Again, this needs to be a polished piece of work.

# APPENDIX U: WRITING PROMPT #21

In an experiment designed to compare the effectiveness of two methods of teaching Italian, 20 students were randomly assigned to each of the methods. The scores on a final exam are to be compared. A summary of the results is given in the following table.

| | *n* | $\bar{\chi}$ | *s* |
|---|---|---|---|
| Method A | 20 | 82 | 12 |
| Method B | 20 | 77 | 14 |

For this problem, do not assume that the two population standard deviations are the same.

a. State the appropriate null and alternative hypotheses for comparing the two methods.
b. Calculate the test statistic for the comparison.
c. What is the approximate distribution of the test statistic under the assumption that the null hypothesis is true?
d. Give an approximate P-value for the significance test.
e. What do you conclude?
f. Draft a four-sentence statement to the Board of Education describing the results of your analysis and recommendation for which method should be employed at the schools in the district.

# APPENDIX V: WRITING PROMPT #22

A quarterback completes 44% of his passes.

a) Explain how you could use a table of random numbers to simulate this quarterback attempting 20 passes.
b) Using your scheme from 6, simulate 20 passes. Using the random digit table, begin on line 149. List the numbers generated and circle the "successes." Calculate the percent of passes completed.
c) What is the probability that the quarterback throws 3 incomplete passes before he has a completion?
d) How many passes can the quarterback expect to throw before he completes a pass?
e) Construct a probability distribution table (out to n = 6) for the number of passes attempted before the quarterback has a completion.
f) Write a four-sentence summary to the football coach of the team that this quarterback plays telling him how he can use this information in a game.

# APPENDIX W: WRITING PROMPT #23

A seed producer certifies that 90% of a certain type of seed will germinate under ideal conditions. A testing agency attempts to germinate 300 of these seeds; 257 germinate.

a. What is the probability that no more than 257 out of 300 seeds would germinate if in fact each seed has probability 90% of germinating?
b. Write a letter to the seed producer explaining if his claim is likely. Make a recommendation to him whether he should keep producing the seed or try a new method. Remember to use statistics in your explanation.

# APPENDIX X: WRITING PROMPT #24

The presence of flu virus is tested by inoculating an egg and seeing if the virus multiplies. Throat swabbings from Pennsylvania American Legion members who died of a mysterious disease after attending their convention were tested this way -- no flu virus was found. A virologist pointed out that it is better to also inoculate some eggs with known flu virus to check that it does grow.

a. Explain why this is a better design for the experiment.
b. Design a problem using sample design. Also, write the answer to the problem as well.

# APPENDIX Y: SCORING RUBRIC FOR PROBLEM-SOLVING PROCESSES

| Level | Problem-Solving Ability | Conceptual Understanding |
|---|---|---|
| 4 | Identifies the goal of the problem or task. Develops a plan that shows an understanding of all components of the problem. Plan is executed with no errors. | Identifies and provides information about major concepts; supplies examples or illustrations with explanations when appropriate. |
| 3 | Identifies the goal of the problem or task. Develops a plan that shows an understanding of the problem but may contain minor errors in executing the plan. | Identifies and provides information about major concepts but may omit minor details. May use examples or illustrations when appropriate but may not effectively relate them to mathematical concepts. |
| 2 | Identifies the goal of the problem or task but misinterprets one or more of the components of the problem. Plan indicates minimal understanding of problem. | Identifies and provides support for major concepts but may have minor errors in logic or understanding. Minor details are ignored or supported with incorrect or flawed thinking. |
| 1 | Does not identify the goal of the problem or task but response shows some evidence of understanding the general nature of the problem. Does not develop a plan. | Does not correctly identify major concepts and information contains errors in logic or understanding. |
| 0 | No evidence of understanding the goal of the task or problem. No attempt to specify or develop a plan. | No attempts are made to identify or provide information about major concepts or the information has no mathematical soundness. |

(Pugalee, 2005, p.131)

# APPENDIX Z: SCORING RUBRIC FOR PROBLEM-SOLVING PROCESSES

| Level | Procedural Understanding | Mathematical Content | Mathematical Reasoning |
|---|---|---|---|
| 4 | Selects and executes appropriate strategies. Representations and algorithms are appropriate. | The mathematics is accurate. All mathematical concepts and ideas are accurately identified. Mathematical terms are used appropriately. | Completely and accurately provides justification for major steps or processes. Defends reasonableness of answer with supporting reasons. |
| 3 | Selects and executes appropriate strategies. Representations and algorithms may have minor errors but do not affect the solution. | The mathematics is accurate. Mathematical concepts and ideas are accurately identified. Mathematical terms are used appropriately, but there may be minor errors. | Accurately provides justification for major steps or processes but lacks clarity or detail. Defends reasonableness of answer but may have minor omissions or errors in describing approach. |
| 2 | Selects appropriate approach, but execution is flawed. Representations and algorithms may be appropriate for the task but are not executed properly. | The mathematics contains minor errors. Mathematical concepts and ideas are identified but with minor errors. There are notable errors in the use of mathematical terms. | Provides justification for most of the steps or processes with no errors. Defends reasonableness of answer but may not develop supporting reasons. |
| 1 | Selects an inappropriate approach or selects the appropriate approach but cannot begin implementation. Representations and algorithms are not appropriate for the task. | The mathematics is mostly inaccurate. Mathematical concepts and ideas are identified with several errors. Mathematical terms are used inappropriately. | Provides some justification for steps or processes but contains numerous errors. Limited or no supporting evidence defending reasonableness of answer. |

APPENDIX Z (CONTINUED)

| | | | |
|---|---|---|---|
| 0 | No evidence of representations or algorithms that would indicate an acceptable approach. | No answer, or mathematics has no relationship to the task. | Does not attempt to provide any justification for steps or processes. |

(Pugalee, 2005, p.131)